Math 30820 Honors Algebra 4
Homework 2

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Do 6 of the following questions. Some questions may be obligatory. Artin a.b.c means chapter a, section b, exercise c. You may use any problem to solve any other problem.

1. (You have to do this problem) Suppose R is a commutative ring with unit and M is an R-module. Define the annihilator of M in R as $\mathrm{Ann}_R(M) = \{r \in R \mid rm = 0, \forall m \in M\}$.
   (a) Show that $\mathrm{Ann}_R(M)$ is an ideal of R.
   (b) Show that if $I \subset \mathrm{Ann}_R(M)$ is an ideal of R then M is naturally an R/I-module.
   (c) What is $\mathrm{Ann}_R(R/I)$?
2. Show that the integral domain $R = \mathbb{C}[X^2, X^3]$ is not *integrally closed*, i.e., that there exists an element $\alpha \in \operatorname{Frac} R$ such that $\alpha \notin R$ and α is integral over R.
3. Let F be a field and $A \in M_{n\times n}(F)$ be a matrix. Recall that in class we defined the following $F[X]$-module M_A: as an abelian group $M_A = F^n$ and scalar multiplication is given by $P(X) \cdot v := P(A)v$ where $P(A) \in M_{n\times n}(F)$ and F^n is interpreted as $M_{n\times 1}(F)$. Suppose $S \in \mathrm{GL}(n, F)$. Show that $M_A \cong M_{SAS^{-1}}$ as $F[X]$-modules.
4. Artin 14.1.4 on page 437.
5. Artin 14.2.3 (a) and from (b) the "only if" part on page 437.
6. Artin 14.2.4 on page 437.
7. Artin 14.7.9 on page 439.
8. Artin 14.8.2 on page 440. (Here the "corresponding linear operator" refers to multiplication by t.)