

Math 40520 Theory of Number

Homework 3

Due Friday, 9/21, in class

Do 5 of the following problems.

1. Exercise 2.13 on page 46 in the textbook.
2. Exercise 2.14 on page 46 in the textbook.
3. Exercise 2.23 on page 47 in the textbook.
4. Exercise 2.31 on page 47 in the textbook.
5. Show that if $n \geq 1$ is an integer then

$$7^{2^n} \equiv 1 + 2^{n+3} \pmod{2^{n+4}}$$

and determine the multiplicative order of 7 modulo 2^m for integers $m \geq 1$.

6. Show that there are infinitely many primes $\equiv 5 \pmod{6}$.
7. Do there exist integers n such that $\varphi(n) = 14$?
8. (Bonus question) For what integers n does $\varphi(n)$ divide n ?