Math 43900 Problem Solving
Fall 2018
Lecture 9 Matrices

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1 Problems

1.1 Determinants, traces, characteristic polynomials and eigenvalues

Easier

1. [Hint: Show that $\det M(t)$ is linear in t, where $M(t)$ the the matrix obtained by adding t to each entry of M..]

2. [Hint: Play around with row and column operations. Perhaps try small values of n..]

3. [Hint: Subtract the first row from all the others, then expand. Or, apply the Putnam 1978 problem..]

Harder

4. [Hint: What rank does the matrix $M(a)$ have? What are its eigenvalues?.]

5. [Hint: Compute the square of the matrix $S = \sum_{i=1}^{r} M_i$..]

1.2 Algebraic operations and linear algebra

Easier

6.

7.

Harder

8. [Hint: Express the conditions as one single matrix multiplication condition..]

9. [Hint: Concoct a square matrix from $\operatorname{Re} M$ and $\operatorname{Im} M$..]

1.3 Extra problems

Easier

10. [Hint: What is the trace?.]

11. [Hint: Double the first row and expand the determinant.]

12. [Hint: What are the eigenvalues of $P(A) = 0$?.]

13. [Hint: If $Xv = \alpha v$ compute $\langle X^t v, v \rangle = \langle v, Xv \rangle$ in two ways..]

14.

15. [Hint: Look at the (quadratic) polynomial $\det(A + BX)$..]

16. [Hint: AG208.]

17.

18.

19.

20.

Harder

21. [Hint: Eigenvalues..]

22. [Hint: Eigenvalues..]

23. [Hint: Use row operations to simplify the matrix..]

24. [Hint: AG213.]

25. [Hint: Eigenvalues.]

26. [Hint: AG217.]

27. [Hint: Use the Exercise 18..]

28. [Hint: Conjugate A to a Jordan canonical form, then things are much easier..]

29. [Hint: Use Exercise 20 and then complete squares..]