Math 43900 Problem Solving
Fall 2018
Lecture 11 Inequalities

Andrei Jorza

1 Problems

1.1 AM-GM, Completing the square, Cauchy-Schwarz, Chebyshev

Easier

1. [Hint: Areas vary quadratically.]
2. [Hint: Use calc 3 if you're up for it, but it's much easier with Cauchy-Schwarz. For the latter, maximize $f(x,y,z)^2$..]
3. [Hint: Apply Cauchy-Schwarz twice..]

Harder

4. [Hint: Apply Cauchy-Schwarz to $n-1$ terms where one is a_i+a_j..]
5. [Hint: Use the idea of Chebyshev..]

1.2 Inequalities in calculus and geometry

Easier

6. [Hint: Compute the area of the triangle in terms of p and r, and then use that $A=\sqrt{p(p-a)(p-b)(p-c)}$..]
7. [Hint: Express the sum in terms of $f(x)$..]
8. [Hint: Calculus.]

Harder

9. [Hint: Show that $f(x,y)+2(x^2+y^2)$ has a minimum..]
10. [Hint: Compare $2P(z)$ with $1+\frac{1}{1-z}$..]

1.3 Miscellaneous

Easier

11. [Hint: Use trig substitutions..]
12. [Hint: Take logs and then use Riemann sums..]

Harder

13. [Hint: Count $k \leq n$ such that $\delta(m)/m$ has a particular value..]

1.4 Extra problems

Easier

14. [Hint: Complete the square..]

15. [Hint: Calculus.]

16. [Hint: What's the case of equality in Cauchy-Schwarz?.]

17. [Hint: Use Riemann sums and Cauchy-Schwarz..]

18. [Hint: You may use the following standard result from honors algebra 3: if $3^k \mid 2^n - 1$ then $2 \cdot 3^{k-1} = \varphi(3^k) \mid n$. Put in abstract algebra language: $(\mathbb{Z}/3^k\mathbb{Z})^\times$ is a cyclic group of order $\varphi(3^k)$ and 2 is a generator. To show this last statement show by induction that $2^{3^t} \equiv -1 + 3^{r+1} \pmod{3^{r+2}}$ and $4^{3^r} \equiv 1 + 3^{r+1} \pmod{3^{r+2}}$..]

Harder

19. [Hint: Show that $x^2 \geq x - 1/4$ and then use AM-GM..]

20. [Hint: Enough to show that $a_k - n \in [-1, 1]$, or equivalently that $(a_k - n)^2 \leq 1$..]

21. [Hint: Write $a_k = x_k - x_{k-1}$ and rewrite the inequality in terms of the a_k..]

22. [Hint: Get rid of sec and use Cauchy-Schwarz.]

23. [Hint: Apply Cauchy-Schwarz to find n. Then play around..]

24. [Hint: $A = \frac{1}{2}bc \sin A$ and $a^2 = b^2 + c^2 - 2bc \cos A$..]

25. [Hint: Sub the equation in the inequalities.]