Math 43900 Problem Solving
Fall 2018
Lecture 12 Functions and functional equations

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1 Problems

1.1 Functional equations and the integers

Easier

1. [Hint: Apply f to the second condition.]

Harder

2. [Hint: By contradiction.]

1.2 Functional equations and algebraic manipulations

Easier

3. [Hint: Substitute x with $1 - 1/x$ a couple oftime..]

Harder

4. [Hint: Treat $x, f(x), f(f(x)), \ldots$ as a linear recurrence..]

5. [Hint: What is the convergence behavior of the sequence $a_0 = x$, $a_{n+1} = a_n^2 + c$?.]

1.3 Functional equations and calculus

Easier

6. [Hint: Start with a few examples..]

7. [Hint: Show that $f^2 + g^2$ is constant first..]

Harder

8. [Hint: Use a trig substitution..]

9. [Hint: Write the functions equations in terms of the difference operator $(\Delta h)(x) = h(x+1) - h(x)$ and use Bezout in the ring $\mathbb{C}[\Delta]$..]

1.4 Extra problems

Easier

10. [Hint: Induction.]

11. [Hint: Apply f to the first equation..]

12. [Hint: Let $x + y = \alpha$ in the functional equation..]

13. [Hint: Play around with special values of x and y..]

14. [Hint: Play around with special values of x and y..]

15. [Hint: (a) Use log, (b) Reduce to (a), (d) divide by xy. .]

Harder

16. [Hint: Use the first part for the second..]

17. [Hint: You know that $f(f(y)) = 1/y$..]

18. [Hint: Plug back the relations from the related problem into the functional equation.]

19. [Hint: Try $y = 2$ and then $x + y = 2$. For the last step try to see if you can make $f(xf(y))$ vanish.]

20. [Hint: Guess!.]

21.

22. [Hint: Compute $f(x/2)$ in terms of $f(x)$ and find a different functional equation satisfied by f..]

23. [Hint: Algebraic manipulation to get something familiar.]