Multiple Choice

1.(6 pts) Let $\mathbf{a} = \langle 1, 2, 0 \rangle$, $\mathbf{b} = \langle 3, 1, -1\rangle$, and let $\mathbf{c} = \text{proj}_{\mathbf{a}}\mathbf{b}$ be the vector projection of $\mathbf{b}$ onto $\mathbf{a}$. Which one of the following vectors is orthogonal to $\mathbf{b} - \mathbf{c}$?

(a) $\langle 0, 1, 1\rangle$ (b) $\langle 2, 1, -1\rangle$ (c) $\langle 1, 2, 0\rangle$

(d) $\langle 2, 1, 0\rangle$ (e) $\langle 1, 0, 1\rangle$

2.(6 pts) Find the radius of the sphere given by the equation

$$x^2 + y^2 + z^2 - 6x + 4z + 7 = 10.$$

(a) 3 (b) 9 (c) -4 (d) 2 (e) 4

3.(6 pts) A particle moves with the position function $\mathbf{r}(t) = \langle t^2, -t, 2\rangle$. Find the normal component of acceleration.

(a) $a_N = \dfrac{2}{\sqrt{1+4t^2}}$ (b) $a_N = 4t$ (c) $a_N = 2$

(d) $a_N = \dfrac{4t}{\sqrt{1+4t^2}}$ (e) $a_N = \sqrt{1+4t^2}$

4.(6 pts) Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 1, 2, 2\rangle$, $\mathbf{b} = \langle 3, 2, 2\rangle$, and $\mathbf{c} = \langle 7, 3, 1\rangle$.

(a) -4 (b) 8 (c) 3 (d) -8 (e) 4

5.(6 pts) Where does the line with parametric equations

$$x = -1 + 3t \qquad y = 2 - 2t \qquad z = 3 + t$$

intersect the plane $3x + y - 4z = -4$?

(a) they do not intersect (b) $(-3, -3, -2)$ (c) $(8, -4, 6)$

(d) $(-10, 8, 0)$ (e) $(0, 0, 1)$

6.(6 pts) Find symmetric equations for the line through the point $(1, -2, -4)$ which is orthogonal to the plane $2x - y + 3z = 18$.

(a) $\dfrac{x+1}{2} = \dfrac{y-2}{1} = \dfrac{z-4}{3}$

(b) $\dfrac{x-1}{2} = \dfrac{-y-2}{-1} = \dfrac{z+4}{3}$

(c) $\dfrac{x-1}{2} = \dfrac{y+2}{-1} = \dfrac{z+4}{3}$

(d) $\dfrac{x-1}{\sqrt{14}} = \dfrac{-y-2}{\sqrt{14}} = \dfrac{z+4}{\sqrt{14}}$

(e) $\dfrac{1+2x}{\sqrt{14}} = \dfrac{-y-2}{\sqrt{14}} = \dfrac{-4+3z}{\sqrt{14}}$

7.(6 pts) Find the position $\mathbf{r}(1)$ of a particle at time $y = 1$ if it has acceleration $\mathbf{a}(t) = e^t\mathbf{i} - 6t\mathbf{k}$, the initial position of the particle is $\mathbf{r}(0) = \langle 1, 0, -1\rangle$ and the initial velocity is $\mathbf{v}(0) = \langle 1, 1, 0\rangle$.

(a) $\mathbf{r}(1) = \langle 1, 0, 1\rangle$

(b) $\mathbf{r}(1) = \langle e, 0, 0\rangle$

(c) $\mathbf{r}(1) = \langle e, 1, -1\rangle$

(d) $\mathbf{r}(1) = \langle e, 1, -2\rangle$

(e) $\mathbf{r}(1) = \langle 0, 1, 2\rangle$

8.(6 pts) Which of these is an equation of the tangent line to the curve

$$\mathbf{r}(t) = \langle t^2 + 2t + 3,\ 4t\cos(t),\ 2e^{3t}\rangle$$

at the point where $t = 0$?

(a) $\langle x, y, z\rangle = \langle 3, 4, 2e\rangle + t\langle 2, 0, 6e\rangle$

(b) $\langle x, y, z\rangle = \langle 3, 0, 2\rangle + t\langle 1, 2, 3\rangle$

(c) $\langle x, y, z\rangle = \langle 3, 0, 2\rangle + t\langle 1, -2, 3\rangle$

(d) $\langle x, y, z\rangle = \langle 3, 4, 2\rangle + t\langle 1, 2, 3\rangle$

(e) $\langle x, y, z\rangle = \langle 3, 0, 2e\rangle + t\langle 2, 4, 6\rangle$

9.(6 pts) Which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = t^2\mathbf{i} - \mathbf{j} + \ln t\,\mathbf{k}$ between the points $(1, -1, 0)$ and $(e^2, -1, 1)$?

(a) $\int_1^{e^2} \sqrt{4t^2 + 1/t^2}\, \mathrm{d}t$

(b) $\int_1^e \sqrt{t^2 + 1 + \ln^2 t}\, \mathrm{d}t$

(c) $\int_0^1 \sqrt{2t + \ln t}\, \mathrm{d}t$

(d) $\int_1^e \sqrt{2t + \ln t}\, \mathrm{d}t$

(e) $\int_1^e \sqrt{4t^2 + 1/t^2}\, \mathrm{d}t$

10.(6 pts) Which one of the following functions has level curves drawn below?

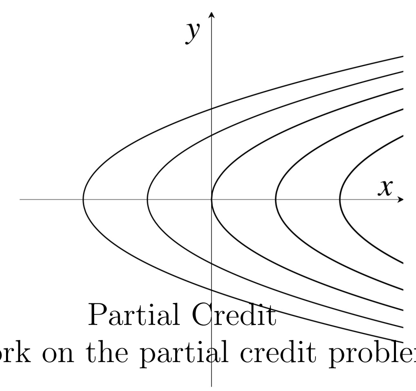
(a) $f(x, y) = y^2 + x$

(b) $f(x, y) = y + x^2$

(c) $f(x, y) = y - x^2$

(d) $f(x, y) = y^2 - x$

(e) $f(x, y) = y^2 - x^2$



Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Find an equation for the line of intersection of the planes $3x - y + z = 0$ and $2x - 3y + z = 0$.

12.(12 pts.) The position function of a moving object is $\mathbf{r}(t) = t^2\mathbf{i} - \mathbf{j} + \ln t\,\mathbf{k}$.

(a) Find the unit tangent vector $\mathbf{T}$, the principal normal vector $\mathbf{N}$, and the bi-normal vector $\mathbf{B}$ at $t = 1$.

(b) Find an equation of the normal plane at $t = 1$.

(c) Find an equation of the osculating plane at $t = 1$.

13.(12 pts.) Find the distance from the point $(-4, 1, 4)$ to the plane containing the points $P(0, 0, 3)$, $Q(1, 1, 3)$, and $R(1, 0, -1)$.

Name: ______________________________

Instructor: ANSWERS

Math 20550, Exam 1, Practice
September 22, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 4 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(•)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(•)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(b)	(c)	(•)	(e)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(•)
10.	(a)	(b)	(c)	(•)	(e)

Please do NOT write in this box.

Multiple Choice	
11.	
12.	
13.	
Extra Points.	4
Total	

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11. Clearly the origin $(0,0,0)$ is on both planes and hence on the intersection line. To get an equation of the line we also need a direction. The line is perpendicular to the normals to both the planes. The normals are $(3,-1,1)$ and $(2,-3,1)$ respectively. So the direction of the line is given by their cross product:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 1 \\ 2 & -3 & 1 \end{vmatrix} = \langle 2,-1,-7\rangle$$

So the equation of the line is:

$$\frac{x}{2} = \frac{y}{-1} = \frac{z}{-7}.$$

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12. At $t = 1$ the position vector is $\mathbf{r}(1) = (1,-1,0)$.

We will need first two derivative at $t = 1$.

$$\mathbf{r}'(t) = 2t\mathbf{i} + \frac{1}{t}\mathbf{k} \text{ and } \mathbf{r}'(1) = 2\mathbf{i} + \mathbf{k}$$

$$\mathbf{r}''(t) = 2\mathbf{i} - \frac{1}{t^2}\mathbf{k} \text{ and } \mathbf{r}'(1) = 2\mathbf{i} - \mathbf{k}$$

(a) We have $\mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{k})$.

Recall that $\mathbf{B} = \frac{\mathbf{r}' \times \mathbf{r}''}{|\mathbf{r}' \times \mathbf{r}''|}$.

Since

$$\mathbf{r}' \times \mathbf{r}'' = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 1 \\ 2 & 0 & -1 \end{vmatrix} = 4\mathbf{j},$$

we have $\mathbf{B} = \mathbf{j}$.

We also have

$$\mathbf{N} = \mathbf{B} \times \mathbf{T} = \mathbf{j} \times \frac{1}{\sqrt{5}}(2\mathbf{i} + \mathbf{k}) = \frac{1}{\sqrt{5}}(2\mathbf{j} \times \mathbf{i} + \mathbf{j} \times \mathbf{k}) = \frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{k}).$$

(b) Normal is plane is ortogonal to $\mathbf{r}'$ and its equation is

$$2(x-1) + z = 0 \text{ or } 2x + z = 2$$

(c) The osculating plane is orthogonal $\mathbf{B}$ and its equation is

$$y + 1 = 0.$$

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13. First we find an equation of the plane containing the points $P(0,0,3)$, $Q(1,1,3)$, and $R(1,0,-1)$. The plain contains vectors $\overrightarrow{\mathbf{PQ}} = \langle 1,1,0\rangle$ and $\overrightarrow{\mathbf{PR}} = \langle 1,0,-4\rangle$, and their crossproduct

$$\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{PR}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 1 & 0 & -4 \end{vmatrix} = \langle -4, 4, -1 \rangle$$

is orthogonal to the plain.

Since the plain contains the point $P = (0,0,3)$ an equation of the plain is

$$-4x + 4y - (z-3) = 0 \text{ or } -4x + 4y - z + 3 = 0$$

Using the distance formula we obtain thea the distance from the point $(-4,1,4)$ to the plain is

$$D = \frac{|-4 \cdot (-4) + 4 \cdot 1 - 4 + 3|}{\sqrt{4^2 + 4^2 + 1^2}} = \frac{19}{\sqrt{33}}.$$