Multiple Choice

1.(6 pts) Let $f(x,y)$ be a function where $(1,3)$ and $(-1,0)$ are critical points. We also know that $f_{xx}(1,3) = 1, f_{x,y}(1,3) = 2, f_{yy}(1,3) = 1$ and $f_{xx}(-1,0) = 2, f_{x,y}(-1,0) = -1, f_{yy}(-1,0) = 3$. Using the second derivative test classify the points $(1,3)$ and $(-1,0)$.

(a) both are local minimums

(b) $(1,3)$ is a saddle point; $(-1,0)$ is a local minimum

(c) $(1,3)$ is a saddle point; $(-1,0)$ is a local maximum

(d) both are saddle points

(e) $(1,3)$ is a local maximum; $(-1,0)$ is a local minimum

2.(6 pts) Use implicit differentiation to find $\partial z/\partial x$ when $xz + z^2 = y$.

(a) $\partial z/\partial x = \dfrac{-z}{x+2z}$

(b) $\partial z/\partial x = \dfrac{y}{x+z}$

(c) $\partial z/\partial x = \dfrac{-x}{2z}$

(d) $\partial z/\partial x = \dfrac{y-z}{x+2z}$

(e) $\partial z/\partial x = \dfrac{y-x}{2z}$

3.(6 pts) Find the directional derivative of $f(x,y) = xe^{-2y}$ at the point $(1,0)$ in the direction $\langle 1,3\rangle$.

(a) $\dfrac{-5}{\sqrt{10}}$ (b) 0 (c) -4

(d) $\dfrac{-1}{2}$ (e) $\sqrt{10}$

4.(6 pts) Let f be the function $f(x, y, z) = \sin(xyz)$. From the point $(1, 1, 0)$ in which direction should one move in order to attain the maximum rate of change.

(a) $\langle 0, 0, 0\rangle$ (b) $\frac{1}{\sqrt{2}}\langle 0, 0, 1\rangle$ (c) $\frac{1}{\sqrt{2}}\langle 1, 1, 0\rangle$ (d) $\langle 1, 1, 1\rangle$ (e) $\langle 0, 0, 1\rangle$

5.(6 pts) Let $f(x, y)$ be a function of $x(s, t) = st$ and $y(s, t) = 2s + t$. If you know that $f_x(1, 3) = 2$ and $f_y(1, 3) = -3$ then what is $\partial f/\partial s$ at when $s = 1$ and $t = 1$?

(a) -1

(b) not enough information to determine the value

(c) 3

(d) -4

(e) 0

6.(6 pts) Find a point on the surface $z = x^2 - y^3$ where the tangent plane is parallel to the plane $x + 3y + z = 0$.

(a) no such point exists (b) $(-1/2, 1, -3/4)$ (c) $(1, 1, 0)$

(d) $(-1/2, 1, 1)$ (e) $(-1/2, 1, -5/2)$

7.(6 pts) Consider the two surfaces $\mathcal{S}_1 : y + z = 4$ and $\mathcal{S}_2 : z = 2x^2 + 3y^2 - 12$. Find the tangent line to the intersection curve of $\mathcal{S}_1$ and $\mathcal{S}_2$ at the point $(1, 2, 2)$.

(a) $\langle x, y, z\rangle = \langle -11t, 4t, -4t\rangle + \langle 1, 2, 2\rangle$

(b) $\langle x, y, z\rangle = \langle -13t, 4t, -4t\rangle + \langle 1, 2, 2\rangle$

(c) $\langle x, y, z\rangle = \langle -11t, 4t, -4t\rangle + \langle -1, -2, -2\rangle$

(d) $\langle x, y, z\rangle = \langle 11t, -4t, 4t\rangle + \langle 1, 2, 2\rangle$

(e) $\langle x, y, z\rangle = \langle -13t, 4t, -4t\rangle + \langle -1, -2, -2\rangle$

8.(6 pts) Find the absolute maximum of the function $f(x, y, z) = xy + \dfrac{z^2}{2}$ under the two constraints $y - 2z = 0$ and $x + z = -1$.

(a) $\dfrac{22}{9}$ (b) $\dfrac{-2}{9}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{2}{9}$ (e) $\dfrac{-1}{2}$

9.(6 pts) Which of the following integrals represents the volume of the solid delimited by $y = 0$, $y = 1$, $x = 0$,$x = 2$, $z = x^2y + y^3$ and $z = 0$.

(a) $\int_0^2 \int_0^1 x^2y + y^3\, dydx$

(b) $\int_0^2 \int_0^1 -x^2y - y^3\, dxdy$

(c) $\int_0^2 \int_0^1 -x^2y - y^3\, dydx$

(d) $\int_0^2 \int_0^1 x^2y + y^3\, dxdy$

(e) $\int_1^2 \int_0^1 x^2y + y^3\, dydx$

10.(6 pts) Compute $\iint_R 24xy\, dA$ where R is the region bounded by $x = 1, x = 2, y = x$, and $y = x^2$.

(a) 62 (b) 128 (c) 64 (d) 48 (e) 81

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Find the absolute maximum and absolute minimum of the function $f(x, y) = x - 3y$ subject to the constraint $x^2 + 2y^2 = 3$.

12.(12 pts.)

Consider the iterated integral $\int_0^2 \int_{y^2}^4 y^3 e^{x^3}\, dxdy$.

(a) Sketch the region of integration.
(b) Rewrite the integral with the order of integration reversed.
(c) Compute the value of the iterated integral.

13.(12 pts.) Determine the absolute maximum and minimum of the function $f(x, y) = x^2y - xy + x$ on the region $0 \leq x \leq 2$, $-2 \leq y \leq 0$.

Name: ______________________

Instructor: ANSWERS

Math 20550, Exam 2
September 30, 2014

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 5 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(•)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(•)
5.	(a)	(b)	(c)	(•)	(e)
6.	(a)	(•)	(c)	(d)	(e)
7.	(a)	(•)	(c)	(d)	(e)
8.	(a)	(b)	(•)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(•)

Please do NOT write in this box.

Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

11. Lagrange system:

$$1 = \lambda 2x$$
$$-3 = \lambda 4y$$
$$x^2 + 2y^2 = 3$$

From the first equation we see that $x \neq 0$, and dividing by $2x$ we get $\lambda = \dfrac{1}{2x}$.

Substituting λ by $\dfrac{1}{2x}$ in the second equation we get $-3 = \dfrac{2y}{x}$, hence $y = -\dfrac{3}{2}x$

Using the constraint we obtain

$$x^2 + 2\frac{9}{4}x^2 = 3 \text{ hence } x^2 = \frac{6}{11}.$$

We have two solutions:

$$x = \frac{\sqrt{6}}{\sqrt{11}}, \quad y = -\frac{3\sqrt{6}}{2\sqrt{11}}$$

and

$$x = -\frac{\sqrt{6}}{\sqrt{11}}, \quad y = \frac{3\sqrt{6}}{2\sqrt{11}}$$

In the first case the value of f is $\dfrac{\sqrt{66}}{2}$, and in the second case the the value of f is $-\dfrac{\sqrt{66}}{2}$.

Answer: the maximum value is $\dfrac{\sqrt{66}}{2}$ and the minimum value is $-\dfrac{\sqrt{66}}{2}$.

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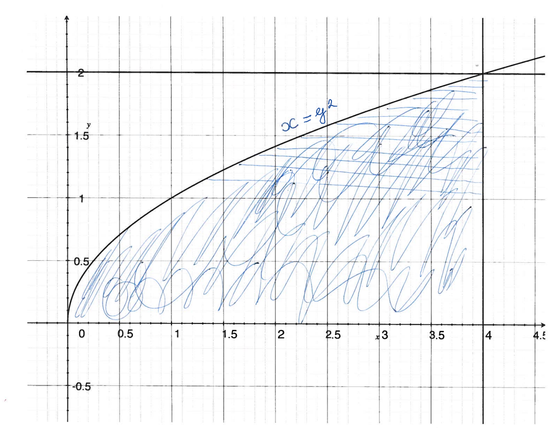
12. For part (a) the region is the following:



For part (b):

$$\int_0^4 \int_0^{\sqrt{x}} y^3 e^{x^3}\, dydx$$

For part (c):

$$\int_0^4 \int_0^{\sqrt{x}} y^3 e^{x^3}\, dydx = \int_0^4 \frac{y^4}{4} e^{x^3}|_0^{\sqrt{x}}\, dx = \frac{1}{4}\int_0^4 x^2 e^{x^3}\, dx$$

$$= \frac{1}{12} e^{x^3}|_0^4 = \frac{1}{12}(e^6 4 - 1)$$

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13. First we find the critical points in the region.

$$f_x : \quad 2xy - y = 0$$
$$f_y : \quad x^2 - x = 0$$

The first equation has two solutions $y = 0$ and $x = \frac{1}{2}$, and the second has solutions $x = 0$ and $x = 1$.
We get the critical points $(0, 0)$ and $(1, 0)$. Both of these points are in the region.

The boundary consists of 4 sides:
(1) $x = 0$, $-2 \le y \le 0$
(2) $x = 2$, $-2 \le y \le 0$
(3) $y = -2$, $0 \le x \le 2$.
(4) $y = 0$, $0 \le x \le 2$.

We analyze each side separately.
Side 1:
$f(0, y) = 0$, so the value of f is constant at 0 on this side.
Side 2:
$f(2, y) = 4y - 2y + 2 = 2y + 2$ This function has no critical points since $f'(y) = 2$ and so we only need to check the endpoints $(2, -2)$ and $(2, 0)$.

Side 3:
$f(x, -2) = -2x^2 + 2x + x = -2x^2 + 3x$. Since $f'(x) = -4x + 3$ it has a critical point at $x = \frac{3}{4}$ which is in the interval $[0, 2]$. On this side we will need to check the points $(\frac{3}{4}, -2)$,$(0, -2)$, and $(2, -2)$.

Side 4:
$f(x, 0) = x$. We have $f'(x) = 1$ and there are no critical points. We need only check $(0, 0)$ and $(2, 0)$.

Now we check all the points we have found.

$$
\begin{gathered}
f(0,0) = 0 \\
f(1,0) = 1 \\
f(2,-2) = -8 + 4 + 2 = -2 \\
f(2,0) = 2 \\
f(\frac{3}{4}, -2) = \frac{-18}{16} + \frac{6}{4} + \frac{3}{4} = \frac{18}{16} = \frac{9}{8} \\
f(0,-2) = 0
\end{gathered}
$$

Answer: The maximum value is 2 and the minimum value is -2.