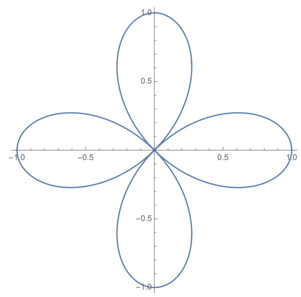
Multiple Choice

1.(6 pts) Evaluate the integral $\iint_D e^{-x^2-y^2}\,dA$ by changing to the polar coordinates, where $D = \{(x,y) | x^2+y^2 \le 1\}$.

(a) $\pi(1-e^{-1})$ (b) $\pi(e^{-1}-1)$ (c) $\pi(1-e)$ (d) $\pi(e-1)$ (e) πe

2.(6 pts) Consider the loop (one leaf) of the 4-leaf rose $r = \cos 2\theta$ which is entirely contained in the first and fourth quadrant.



If this region has density $\rho(x,y) = x^2+y^2$ then which of the following integrals is the moment about the y-axis.

(a) $M_y = \int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} xr^3\,dr\,d\theta$

(b) $M_y = \int_{-\pi/2}^{\pi/2}\int_0^{\cos 2\theta} r^4\cos\theta\,dr\,d\theta$

(c) $M_y = \int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} r^4\cos\theta\,dr\,d\theta$

(d) $M_y = \int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} r^3\cos\theta\,dr\,d\theta$

(e) $M_y = \int_{-\pi/2}^{\pi/2}\int_0^{\cos 2\theta} r^3\cos\theta\,dr\,d\theta$

3.(6 pts) Evaluate $\int\int\int_E zy\,dV$, where

$$E = \{(x,y,z) \,|\, 0 \le x \le 2, \quad 0 \le y \le \sqrt{4-x^2}, \quad 0 \le z \le x\}.$$

(a) 4 (b) 2 (c) 1 (d) $\frac{16}{15}$ (e) $\frac{1}{2}$

4.(6 pts) A solid E lies within the cylinder $x^2+y^2=1$, below the plane $z=4$ and above $z = 1-x^2-y^2$. The density at any point is equal to its distance from the z axis. Find an integral that computes the mass of E.

(a) $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r^2\,dz\,dr\,d\theta$

(b) $\int_0^{2\pi}\int_0^1\int_4^{1-r^2} r^2\,dz\,dr\,d\theta$

(c) $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r\,dz\,dr\,d\theta$

(d) $\int_0^{2\pi}\int_0^1\int_{4-r^2}^1 r^2\,dz\,dr\,d\theta$

(e) $\int_0^{2\pi}\int_0^1\int_{1-r^2}^1 r^2\,dz\,dr\,d\theta$

5.(6 pts) Let E be the region between the spheres $x^2+y^2+z^2=z$ and $x^2+y^2+z^2=2z$. Which of the following represents $\int\int\int_E (x^2+y^2)\;dV$ in spherical coordinates?

(a) $\int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho^4\sin(\phi)\,d\rho\,d\phi\,d\theta$

(b) $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\theta)}^{2\cos(\theta)} \rho^4\sin^3(\theta)\,d\rho\,d\phi\,d\theta$

(c) $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$

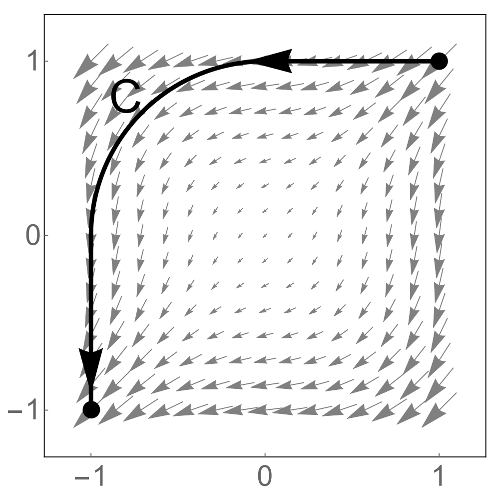
(d) $\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$

(e) $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos(\phi)}^{2\cos(\phi)} \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$

6.(6 pts) Find $\int_C 2xy^3 ds$, where C is the upper half of the circle $x^2+y^2=4$.

(a) 2π (b) 4 (c) 0 (d) 8 (e) 4π

7.(6 pts) The following figure shows a vector field $\mathbf{F}$ on $\mathbb{R}^2$ and a curve C from $(1,1)$ to $(-1,-1)$.



Which of the following statements **must be true** about the line integral of $\mathbf{F}$ over C?

(a) It must be negative.

(b) It must be positive.

(c) It must be zero.

(d) It must be 1.

(e) It is impossible to tell from the image.

8.(6 pts) Calculate $\int_C y\,dx + 4x\,dy$ where C is the curve $\mathbf{r}(t) = \langle t^2, t\rangle$, $0 \le t \le 1$.

(a) -2 (b) 4 (c) $\frac{4}{3}$ (d) -4 (e) 2

9.(6 pts) Which one of the following vector fields is conservative?

(a) $\mathbf{F} = (3x^2 + ye^{xy})\mathbf{i} + (9y^8 + xe^{xy})\mathbf{j}$

(b) None of these vector fields are conservative.

(c) $\mathbf{F} = \big(\sin(y) + 2x\big)\mathbf{i} + \sin(y)\mathbf{j}$

(d) $\mathbf{F} = (3x^2 + xe^{xy})\mathbf{i} + (9y^8 + ye^{xy})\mathbf{j}$

(e) $\mathbf{F} = x\mathbf{i} + x\mathbf{j}$

10.(6 pts) Using the Fundamental Theorem of Line Integrals, evaluate

$$\int_C \big(e^x y + x^2\big)dx + \big(e^x + \cos(y)\big)dy$$

where C is any smooth curve from $(1,0)$ to $(0,\pi)$.

(a) $\frac{2}{3}$ (b) $\pi - \frac{1}{3}$ (c) 0 (d) π (e) $-\pi$

Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(12 pts.) Find $\bar{z}$, the z coordinate of the center of mass, for the solid S bounded by paraboloid $z = x^2 + y^2$ and the plane $z = 1$ if S has constant density 1 and the total mass $\dfrac{\pi}{2}$.

12.(12 pts.) Use the transformation $x = u^2$ and $y = v^2$ to find the area of the region bounded by the curves $\sqrt{x} + \sqrt{y} = 1$, x-axis and y-axis.

13.(12 pts.) Let C be the helix given by the equation $\mathbf{r}(t) = \langle \cos t, \sin t, 8t \rangle$, $0 \leq t \leq \dfrac{\pi}{4}$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $\mathbf{F} = \langle x^2, -xy, 0 \rangle$.

Name: ______________________________

Instructor: ANSWERS

Math 20550. Exam 3, Practice Exam
November 7, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 5 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(•)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(•)	(d)	(e)
7.	(a)	(•)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(•)
9.	(•)	(b)	(c)	(d)	(e)
10.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

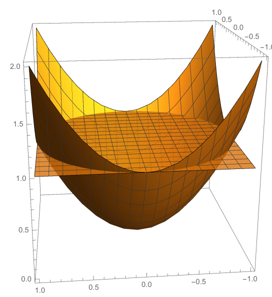
Multiple Choice	________
11.	________
12.	________
13.	________
Extra Points.	4
Total:	________

11. Let S be the solid in the problem. We use cylindrical coordinates to represent S. Thus we set

$$\begin{aligned} x &= r\cos\theta, \\ y &= r\sin\theta, \\ z &= z. \end{aligned}$$

Then the solid

$$S = \{0 \le \theta \le 2\pi, 0 \le r \le 1, r^2 \le z \le 1\}.$$



The z coordinate of the center of the mass is computed by

$$\overline{z} = \frac{\int\int\int_S z\rho dV}{m},$$

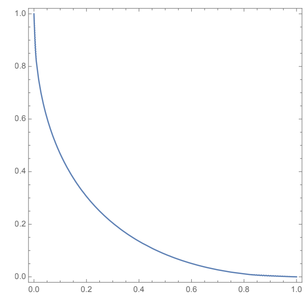
where $m = \dfrac{\pi}{2}$ and $\rho = 1$. So we only need to compute the integral.

$$\begin{aligned} \int\int\int_S z\rho dV &= \int_0^{2\pi}\int_0^1\int_{r^2}^1 zrdzdrd\theta \\ &= \frac{1}{2}\int_0^{2\pi}\int_0^1 (1-r^4)rdrd\theta \\ &= \frac{1}{2}\int_0^{2\pi}\int_0^1 (r-r^5)drd\theta \\ &= \frac{1}{2}\int_0^{2\pi}(\frac{1}{2}-\frac{1}{6})d\theta = \frac{\pi}{3}. \\ &= \frac{1}{2}\cdot\frac{1}{3}\cdot 2\pi = \frac{\pi}{3}. \end{aligned}$$

Hence

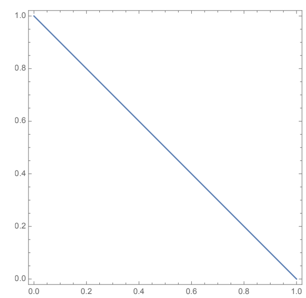
$$\overline{z} = \frac{\frac{\pi}{3}}{\frac{\pi}{2}} = \frac{2}{3}.$$

12. Let D be the region in the problem.



Under the transformation, we get the transform of D as

$$S = \{u + v \leq 1, u \geq 0, v \geq 0\}.$$



The Jacobian of the transformation is

$$\frac{\partial(x, y)}{\partial(u, v)} = 4uv.$$

Hence the area is

$$\begin{aligned}\iint_D dA &= \int_0^1 \int_0^{1-u} |\frac{\partial(x, y)}{\partial(u, v)}| dvdu, \\ &= \int_0^1 \int_0^{1-u} 4uvdvdu \\ &= \int_0^1 2u(1-u)^2 du \\ &= \frac{1}{6}\end{aligned}$$

13.

$$\begin{aligned}\int_C \mathbf{F}\cdot d\mathbf{r} &= \int_C x^2\,dx - xy\,dy\\ &= \int_0^{\frac{\pi}{4}} \cos^2 t(-\sin t)dt - \cos t\sin t\cos t dt\\ &= -2\int_0^{\frac{\pi}{4}} \cos^2 t\sin t dt\end{aligned}$$

Use the substitution

$$u = \cos t, \quad du = -\sin t.$$

$$\begin{aligned}-2\int_0^{\frac{\pi}{4}} \cos^2 t\sin t dt &= \\ &= \frac{2}{3}u^3\Big|_1^{\frac{\sqrt{2}}{2}}\\ &= \frac{2}{3}(\frac{\sqrt{2}}{4} - 1).\end{aligned}$$

Hence

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \frac{2}{3}(\frac{\sqrt{2}}{4} - 1).$$