Multiple Choice

1.(6 pts) If the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\text{Comp}_{\mathbf{a}}\,\mathbf{b} = 1$, what is $\text{Comp}_{2\mathbf{a}}\,3\mathbf{b}$?

(a) 2 (b) 5 (c) $\dfrac{3}{2}$ (d) 6 (e) 3

2.(6 pts) Which of the following expressions gives the length of the curve defined by $\mathbf{r}(t) = \ln(t)\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}$ between the points $(0, -1, 1)$ and $(1, -e, e^2)$?

(a) $\displaystyle\int_1^e \sqrt{\ln^2(t) + t^2 + t^4}\, \mathrm{d}t$ (b) $\displaystyle\int_1^{e^2} \sqrt{1/t + 1 + 4t^2}\, \mathrm{d}t$

(c) $\displaystyle\int_1^e \sqrt{1/t^2 + 1 + 4t^2}\, \mathrm{d}t$ (d) $\displaystyle\int_1^e \sqrt{1/t - 1 + 2t}\, \mathrm{d}t$

(e) $\displaystyle\int_1^e \sqrt{\ln(t) - t + t^2}\, \mathrm{d}t$

3.(6 pts) Find the distance from the point $(1, 2, 3)$ to the plane $x + 2y - 2z = -7$.

(a) $\sqrt{3}$ (b) 1 (c) 6 (d) 2 (e) $\sqrt{6}$

4.(6 pts) Suppose the position function $\mathbf{r}(t) = \langle t^3/3, t^2/2, t\rangle$. Find the normal component of the acceleration vector at $t = 1$.

(a) $a_N = \sqrt{2}$ (b) $a_N = \sqrt{3}$ (c) $a_N = \sqrt{5}$

(d) $a_N = 1$ (e) $a_N = 0$

5.(6 pts) Find the area of the triangle with vertices $(4, 2, 2)$, $(3, 3, 1)$ and $(5, 5, 1)$.

(a) 0 (b) 4 (c) $\sqrt{3}$ (d) $\sqrt{6}$ (e) 2

6.(6 pts) The two curves below intersect at the point $(1, 4, -1) = \mathbf{r}_1(0) = \mathbf{r}_2(1)$. Find the cosine of the angle of intersection

$$\mathbf{r}_1(t) = e^{3t}\mathbf{i} + 4\sin\left(t + \frac{\pi}{2}\right)\mathbf{j} + (t^2 - 1)\mathbf{k}$$
$$\mathbf{r}_2(t) = t\mathbf{i} + 4\mathbf{j} + (t^2 - 2)\mathbf{k}$$

(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) 0 (d) $\frac{e}{\sqrt{e^2 + 4}}$ (e) 3

7.(6 pts) Find the vector equation of the line passing through the point $(1, 1, 1)$ and $(1, 2, 3)$

(a) $\langle x, y, z\rangle = \langle 1, 1, 1\rangle + t\langle 0, 1, 2\rangle$

(b) $\langle x, y, z\rangle = \langle 0, 1, 2\rangle + t\langle 1, 2, 3\rangle$

(c) $\langle x, y, z\rangle = \langle 1, 1, 1\rangle + t\langle 0, 0, 1\rangle$

(d) None of the above

(e) $\langle x, y, z\rangle = \langle 1, 1, 1\rangle + t\langle 1, 2, 3\rangle$

8.(6 pts) Consider a helix curve

$$\mathbf{r}(t) = \langle \cos t - 1, \sin t, t\rangle.$$

Find the equation of the osculating plane of the curve at the point $(0, 0, 0)$

(a) $x = 0$

(b) $-y + z = 0$

(c) None of the above

(d) $y + z = 0$

(e) $x + y + z = 0$

9.(6 pts) Given a space curve

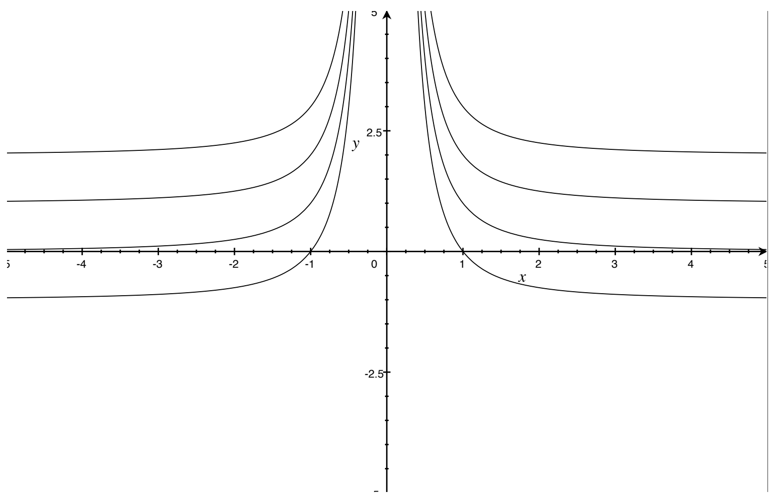
$$\mathbf{r}(t) = \langle 2\cos t, e^t, t\rangle.$$

Which of the following points is in the tangent line of the curve at the point $(2, 1, 0)$?

(a) $(1, 1, 1)$ (b) $(2, 1, 1)$ (c) $(1, 2, 0)$

(d) $(2, 2, 1)$ (e) $(0, 1, 2)$

10.(6 pts) Which of the following functions has this contour map



(a) $f(x, y) = xy$ (b) $f(x, y) = y - \dfrac{xy - 1}{x}$

(c) $f(x, y) = y - \dfrac{1}{x^2}$ (d) $f(x, y) = \dfrac{1}{x}$

(e) $f(x, y) = \dfrac{1}{x^2}$

11.(6 pts) Given three points $P(2, 0, 2)$, $Q(1, 1, 0)$ and $R(1, 2, 3)$.
(a) Find an equation of the plane through P, Q and R.
(b) Find an equation of the line through the point $(1, 1, 1)$ perpendicular to the plane in part (a).

12.(6 pts) (a)Find an equation for the line of intersection of the planes $x - 3y + 2z = 0$ and $2x - 3y + z = 0$.

(b) Does the line from part (a) intersect the line with equations $x = 1 + t$, $y = 3 - t$, $z = 1 + t$? If so, where do they intersect?

13.(6 pts) A particle has the acceleration

$$\mathbf{a}(t) = 2\mathbf{j} + 6t\mathbf{k}.$$

At the time $t = 0$, the particle's position is at the origin and its velocity is $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$. Find the position function $\mathbf{r}(t)$ of the particle.

Name: ________________________

Instructor: ANSWERS

Math 20550, Exam 1
February 17, 2015

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 6 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points. You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(•)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(•)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(•)	(e)
6.	(a)	(•)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(•)	(e)
10.	(a)	(b)	(•)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	
11.	
12.	
13.	
Extra Points.	4
Total	

11.

$$\mathbf{PQ} = (2,0,2) - (1,1,0) = \langle 1,-1,2\rangle$$
$$\mathbf{RQ} = (1,2,3) - (1,1,0) = \langle 0,1,3\rangle$$

So for part (a) we compute $\mathbf{PQ} \times \mathbf{RQ}$ which is $\langle -3-2, -(3-0), 1\rangle = \langle -5,-3,1\rangle$. So the equation of the plane through P, Q, and R is

$$\langle -5,-3,1\rangle \cdot \langle x,y,z\rangle = \langle -5,-3,1\rangle \cdot \langle 1,1,0\rangle$$
$$-5x - 3y + z = -8.$$

For part (b) we use the computations from part (a) to see that the equation of the line should be

$$\langle x,y,z\rangle = \langle 1,1,1\rangle + t\langle -5,-3,1\rangle.$$

12. For part (a) we take the cross product of the normal vectors of each plane.

$$\langle 1,-3,2\rangle \times \langle 2,-3,1\rangle = \langle -3+6, -(1-4), -3+6\rangle = \langle 3,3,3\rangle.$$

This is the direction of the line. So now we just need a point in the intersection of both planes, since both are of the form $ax+by+cz=0$ we can easily see that $(0,0,0)$ is in the intersection. So the equation for the line of intersection is

$$\langle x,y,z\rangle = t\langle 3,3,3\rangle.$$

For part (b) to see if this line intersects the given line, we must solve a system of equations. Changing the paramter in the given line to s we get

$$1+s = 3t$$
$$3-s = 3t$$
$$1+s = 3t$$

Solving the first equation for s to get $s = 3t-1$ and plugging into the 2nd equation to get $3-(3t-1) = 3t$ we can solve to see $t = 2/3$. Which would mean that $s = 1$. Checking these values in the equation we see that this is a solution $1+2 = 3(2/3)$. So using either the $t = 2/3$ or $s = 1$ value we can compute that the intersection point is $(2,2,2)$.

13. To find $\mathbf{v}(t)$ we integrate $\mathbf{a}(t)$, to get

$$\mathbf{v}(t) = \langle c, 2t+d, 3t^2+e\rangle$$

where c, d, and e are constants.

To find the constants we evaluate at 0. To see that $\mathbf{v}(0) = \langle c,d,e\rangle$ which should equal $\langle 1,0,1\rangle$. So $c=1$, $d=0$, and $e=1$.

So now $\mathbf{v}(t) = \langle 1, 2t, 3t^2+1\rangle$.

To find $\mathbf{r}(t)$ we repeat this process,

$$\mathbf{r}(t) = \langle t + c, t^2 + d, t^3 + t + e \rangle.$$

Now using the intial data we have that

$$\langle 0, 0, 0 \rangle = \mathbf{r}(0) = \langle c, d, e \rangle.$$

So $c = 0$, $d = 0$, and $e = 0$.

So the final answer is $\mathbf{r}(t) = \langle t, t^2, t^3 + t \rangle$.