

Multiple Choice

1.(6 pts) Find the absolute maximum and minimum of  $f(x, y) = 4y + x^2 - 2x + 1$  on the closed triangular region with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(0, 2)$ .

- (a) maximum value = 9, minimum value = 0
- (b) maximum value = 4, minimum value = 0
- (c) maximum value = 8, minimum value = 1
- (d) maximum value = 1, minimum value = 0
- (e) maximum value = 10, minimum value =  $-1$

2.(6 pts) Find the equation of the tangent plane to the surface  $xz + \ln(2x + y) = 5$  at the point  $(-1, 3, -5)$ .

- (a)  $3x + y - z - 5 = 0$
- (b)  $-4x + y - z - 4 = 0$
- (c)  $4x - y + z + 12 = 0$
- (d)  $5x - y + z + 13 = 0$
- (e)  $-3x + y - z - 11 = 0$

3.(6 pts) If  $z = f(x, y)$ , where  $f$  is differentiable, and  $x = g(t), y = h(t), g(1) = 3, h(1) = 4, g'(1) = -2, h'(1) = 5, f_x(3, 4) = 7$  and  $f_y(3, 4) = 6$ . Find  $dz/dt$  when  $t = 1$ .

- (a) 13
- (b) 44
- (c) 32
- (d) 23
- (e) 16

4.(6 pts) Find the directional derivative of the function  $f(x, y) = x^2 + y^3$  at the point  $(2, 1)$  in the direction  $\langle 1, 1 \rangle$

- (a)  $\frac{3}{\sqrt{2}}$
- (b) 7
- (c) None of the above
- (d)  $\frac{7}{\sqrt{2}}$
- (e) 3

**5.**(6 pts) For a function  $f(x, y)$ , suppose that  $f_{xx} = x^2$  and  $D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = x^2y^2 - 2$ . Which is true for the points  $P(1, 1)$  and  $Q(1, 2)$  where  $P$  and  $Q$  are critical points of  $f$ .

- (a)  $P$  is a local min and  $Q$  is a local max.
- (b)  $P$  is a local max and  $Q$  is a local min.
- (c)  $P$  is a saddle point and  $Q$  is a local min.
- (d) None of the above
- (e)  $P$  is a saddle point and  $Q$  is a local max.

**6.**(6 pts) What is the equation of the tangent line to the curve of intersection between the two surfaces defined by  $z = x^2 + y^2$  and  $x^2 + 2y^2 + z^2 = 7$  at the point  $(-1, 1, 2)$ .

- (a)  $\langle x, y, z \rangle = \langle -1, 1, 2 \rangle + t\langle 1, 2, 1 \rangle$
- (b)  $\langle x, y, z \rangle = \langle -1, 1, 2 \rangle + t\langle 12, 10, -4 \rangle$
- (c)  $\langle x, y, z \rangle = \langle -1, 1, 2 \rangle + t\langle -2, 2, 1 \rangle$
- (d) None of the above
- (e)  $\langle x, y, z \rangle = \langle -1, 1, 2 \rangle + t\langle -2, 4, 4 \rangle$

7.(6 pts) Find the maximum rate of change of  $f(x, y) = 3e^{xy}$  at the point  $(2, 0)$  and the direction in which it occurs.

- (a) Rate of change =  $\sqrt{3}$  in the direction  $\langle 1, 0 \rangle$
- (b) Rate of change = 36 in the direction  $\langle -1, 0 \rangle$
- (c) Rate of change = 3 in the direction  $\langle 1, 1 \rangle$
- (d) Rate of change =  $\sqrt{6}$  in the direction  $\langle 1, -1 \rangle$
- (e) Rate of change = 6 in the direction  $\langle 0, 1 \rangle$

8.(6 pts) Find absolute maximum and minimum of  $3x - y - 3z$  subject to the constraints  $x + y - z = 0$  and  $x^2 + 2z^2 = 6$ .

- (a) Max=12, Min=-12    (b) Max=6, Min=-1    (c) Max=15, Min=5
- (d) Max= $3\sqrt{5}$ , Min=0    (e) Max=5, Min= $-3\sqrt{5}$

9.(6 pts) Evaluate the iterated integral

$$\int_0^2 \int_y^{2y} 2xy \, dx \, dy.$$

- (a) 5                      (b) 3                      (c) 12                      (d) 4                      (e) 2

10.(6 pts) Which integral represents the volume of the solid below the plane  $x + y + z = 3$  and over the rectangle  $[0, 2] \times [0, 1]$ .

- (a)  $\int_0^1 \int_0^2 3 - x - y \, dy \, dx$                       (b)  $\int_0^1 \int_0^2 x + y + z \, dy \, dx$
- (c)  $\int_0^2 \int_0^1 x + y + z \, dy \, dx$                       (d)  $\int_0^2 \int_0^1 1 \, dy \, dx$
- (e)  $\int_0^2 \int_0^1 3 - x - y \, dy \, dx$

Partial Credit

You must show your work on the partial credit problems to receive credit!

**11.**(12 pts) Find all critical points of  $f(x, y) = x^3 - xy + y^2/2$  and classify them using the second derivative test.

**12.**(12 pts) Use Lagrange Multipliers to find extrema values of the function  $f(x, y) = 2x^3 - y^3$  subject to the constraint  $x^2 + y^2 = 5$ .

**13.**(12 pts) Find the volume of the solid that lies under the graph of  $f(x, y) = xe^{xy}$  and above the rectangle  $R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$ .

Name: \_\_\_\_\_

Instructor: ANSWERS

**Math 20550, Old Exam 2**

**October 26, 2017**

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 minutes..
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 7 pages of the test.
- Each multiple choice question is 6 points, each partial credit problem is 12 points.  
You will receive 4 extra points.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!					
1.	(●)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(●)
3.	(a)	(b)	(c)	(d)	(●)
4.	(a)	(b)	(c)	(●)	(e)
5.	(a)	(b)	(●)	(d)	(e)
6.	(a)	(●)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(●)
8.	(●)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(●)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(●)

**Please do NOT write in this box.**

Multiple Choice \_\_\_\_\_

11. \_\_\_\_\_

12. \_\_\_\_\_

13. \_\_\_\_\_

Extra Points. 4 \_\_\_\_\_

Total: \_\_\_\_\_

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**11.** First we must find critical points, so we want to solve the system of equations:

$$f_x = 3x^2 - y = 0$$

$$f_y = -x + y = 0.$$

We get that  $y = 3x^2$  from the first equation and then plugging into the 2nd equation we get  $-x + 3x^2 = 0$  and so  $x = 0$  or  $-1 + 3x = 0$  so  $x = 0$  or  $x = 1/3$ .

When  $x = 0$ , we get that  $y = 0$  so we get the critical point  $(0, 0)$ . When  $x = 1/3$  we get that  $y = 1/3$  so we get the critical point  $(1/3, 1/3)$ .

Now we apply the 2nd derivative test,  $f_{xx} = 6x$ ,  $f_{xy} = -1$ ,  $f_{yy} = 1$ . So  $D = 6x - 1$ . At  $(0, 0)$ ,  $D = -1$  so  $(0, 0)$  is a saddle point. At  $(1/3, 1/3)$ ,  $D = 2$  and  $f_{xx} = 3$  so  $(1/3, 1/3)$  is a local minimum.

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**12.** The Lagrange system of equations is :

$$6x^2 = 2x\lambda$$

$$-3y^2 = 2y\lambda$$

$$x^2 + y^2 = 5$$

Case 1: Assume  $x \neq 0$  and  $y \neq 0$ . Then we can divide the first two equations by  $2x$  and  $y$  respectively, to get  $3x = \lambda$  and  $-3y = 2\lambda$ . Solving for  $x$  and  $y$  in terms of  $\lambda$  we get  $x = \lambda/3$  and  $y = 2\lambda/-3$ . Plugging these into the third equation we get that  $\lambda^2/9 + 4\lambda^2/9 = 5$  which simplifies to  $5\lambda^2/9 = 5$  and so  $\lambda^2 = 9$  or  $\lambda = \pm 3$ .

So when  $\lambda = 3$  we get  $x = 1$ , and  $y = -2$  so we must check the point  $(1, -2)$ .

When  $\lambda = -3$  we get  $x = -1$  and  $y = 2$  and so we must check the point  $(-1, 2)$ .

Case 2: Assume  $x = 0$ . Then the first equation will be satisfied for any  $\lambda$  value. And the third equation gives us that  $y = \pm\sqrt{5}$ . (The 2nd equation will give us that  $\lambda = -15/2\sqrt{5}$  but this is irrelevant.). So we must check the points  $(0, \sqrt{5})$  and  $(0, -\sqrt{5})$ .

Case 3: Assume  $y = 0$  as in case 2 we will get that  $x = \pm\sqrt{5}$  from equation 3, so we need to check the points  $(\sqrt{5}, 0)$  and  $(-\sqrt{5}, 0)$ .

Now we check all the points found in each of the cases:

$$f(1, -2) = 2 - (-8) = 10$$

$$f(-1, 2) = -2 - 8 = -10$$

$$f(0, \sqrt{5}) = 0 - 5\sqrt{5} = -5\sqrt{5}$$

$$f(0, -\sqrt{5}) = 0 - (-5\sqrt{5}) = 5\sqrt{5}$$

$$f(\sqrt{5}, 0) = 10\sqrt{5} - 0 = 10\sqrt{5}$$

$$f(-\sqrt{5}, 0) = -10\sqrt{5} - 0 = -10\sqrt{5}$$

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So the maximum value is  $10\sqrt{5}$  and the minimum value is  $-10\sqrt{5}$ .

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**13.** We want to compute the integral  $\int_0^1 \int_0^1 xe^{xy} dy dx$  (as integrating with respect to  $x$  first will be less desirable).

So we get

$$\int_0^1 e^{xy}|_0^1 dx = \int_0^1 e^x - 1 dx = e^x - x|_0^1 = e - 1 - (1 - 0) = e - 2$$