Math 30810 Honors Algebra 3
Homework 1

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Due Wednesday, September 11

1. Artin 1.9 on page 32.
2. Artin 1.13 on page 32. [Hint: Do you know a good formula for $\frac{1}{1+x}$ where x is a real number with $|x| < 1$?]
3. Suppose A and B are two $n \times n$ matrices with complex entries and X is an indeterminate variable.
   (a) If B is invertible show that $\det(XI_n - AB) = \det(XI_n - BA)$ as degree n monic polynomials in X.
   (b) Show that $\det(XI_n - AB) = \det(XI_n - BA)$ even if B is not invertible. [Hint: Apply part (a) to $B + aI_n$ for a suitable complex number a.]
4. If $A, B \in M_{n\times n}(\mathbb{R})$ we say that $A \sim_{\mathbb{R}} B$ if there exists an invertible matrix $S \in M_{n\times n}(\mathbb{R})$ such that $A = SBS^{-1}$. Similarly, we say $A \sim_{\mathbb{C}} B$ if there exists an invertible matrix $S \in M_{n\times n}(\mathbb{C})$ such that $A = SBS^{-1}$. Show that if $A \sim_{\mathbb{C}} B$ then $A \sim_{\mathbb{R}} B$. [Hint: Use the real and imaginary parts.]
5. Suppose $*$ is an associative composition law on a set S. Show that for any $a_1, \ldots, a_n \in S$ the value of $a_1 * a_2 * \cdots * a_n$ does not depend on the order in which we evaluate the compositions.