Math 30810 Honors Algebra 3
Homework 2

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Due Wed, September 18

Do 5 problems.

1. Recall from class that if S is the set of smooth functions defined on some open neighborhood of 0 in $\mathbb{R}$ then we have an equivalence relation $\sim$ that identifies two functions that agree on some open neighborhood of 0. Define a natural multiplication function on $S/\sim$. For this multiplication, determine the identity and the invertible elements.
2. Artin 2.1.1 on page 69.
3. Artin 2.2.2 on page 69.
4. Artin 2.2.4 on page 70.
5. Artin 2.2.6 on page 70.
6. Let B be the subset of $\mathrm{GL}_n(\mathbb{R})$ consisting of upper-triangular matrices. Show that B is a subgroup of $\mathrm{GL}_n(\mathbb{R})$.
7. Let T be the subset of $\mathrm{GL}_n(\mathbb{R})$ consisting of diagonal matrices. Show that T is a subgroup of $\mathrm{GL}_n(\mathbb{R})$.
8. Show that the set of matrices

$$H = \left\{ \begin{pmatrix} 1 & x_1 & x_2 & \dots & x_{n-1} & x_n & z \\ 0 & 1 & 0 & 0 & \dots & 0 & y_1 \\ 0 & 0 & 1 & 0 & \dots & 0 & y_2 \\ & & & \ddots & & & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 & y_n \\ 0 & 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \mid x_1, \dots, x_n, y_1, \dots, y_n, z \in \mathbb{R} \right\}$$

forms a subgroup of $\mathrm{GL}_{n+2}(\mathbb{R})$. It is called the Heisenberg group.

9. For a matrix $A \in M_{n\times n}(\mathbb{R})$, let A^t be the transpose matrix, so that the ij-entry of A^t is the ji entry of A. Prove that if $A \in \mathrm{GL}_n(\mathbb{R})$, then $(A^{-1})^t = (A^t)^{-1}$. [Hint: recall that for matrices A and B in $M_{n\times n}(\mathbb{R})$, $(AB)^t = B^t A^t$.]