Math 30810 Honors Algebra 3
Homework 4

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Due Wednesday, October 2

Do 5.

1. Artin 2.5.5
2. Artin 2.6.7
3. Artin 2.6.9. Here if $(G, \cdot_G)$ is a group then the "opposite" group is defined to be $(G^\circ, \cdot_{G^\circ})$ where the elements are the same $G^\circ = G$ but the multiplication is opposite: $g \cdot_{G^\circ} h = h \cdot_G g$.
4. Artin 2.8.4.
5. Artin 2.11.4.
6. Show that every finite subgroup of $\mathbb{C}^\times$ is of the form $\mu_n = \{z \in \mathbb{C} \mid z^n = 1\}$ for some $n \geq 1$.
7. Show that if G is a group of order 4 then either it is cyclic or it is isomorphic to the Klein 4-group V.
8. (This is a useful problem, in the textbook it's 2.5.6) For $1 \leq i, j \leq n$ consider the matrix $E_{ij} \in M_{n\times n}(\mathbb{C})$ with 1 in position ij and 0s everywhere else.
   (a) For $i \neq j$ show that $I_n + E_{ij} \in \mathrm{GL}_n(\mathbb{C})$.
   (b) For a general matrix $X \in \mathrm{GL}_n(\mathbb{C})$ compute XE_{ij} and $E_{ij}X$ and show that the only invertible matrices that commute with every other invertible matrix are the scalar ones.