Math 30810 Honors Algebra 3
Homework 9

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Due Wednesday, November 13

Do 5.

1. Show that the matrices $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ are conjugate in $\mathrm{GL}(2, \mathbb{R})$ but not conjugate in $\mathrm{SL}(2, \mathbb{R})$.

2. Let p be a prime and $n \geq 2$ an integer. Show that the center of $\mathrm{SL}_n(\mathbb{F}_p)$ has order $(n, p-1)$. What is the order of $\mathrm{PSL}_n(\mathbb{F}_p)$? (You may assume the center consists of scalar matrices.)

3. For a matrix $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}(2, \mathbb{R})$ and $z \in \mathbb{C}$ define, if possible, $g \cdot z = \dfrac{az+b}{cz+d}$.

   (a) Show that $\mathrm{Im}(g \cdot z) = \dfrac{\det(g)\,\mathrm{Im}(z)}{|cz+d|^2}$.

   (b) Show that $g \cdot z$ defined an action of the subgroup $\mathrm{GL}(2, \mathbb{R})^+$ of matrices with positive determinant on the set $\mathcal{H} = \{z \in \mathbb{C} |\, \mathrm{Im}\, z > 0\}$.

   (c) Compute the stabilizers $\mathrm{Stab}(i)$ and $\mathrm{Stab}(\zeta_3)$.

   (d) (Optional) Show that this action is transitive, i.e., all of $\mathcal{H}$ is one orbit.

4. Artin 6.7.3 on page 190.

5. Artin 6.7.7 on page 191.

6. Artin 6.8.1 on page 191.

7. Artin 6.M.7 on page 194. (Careful: what Artin calls D_3 in part (a) we called D_6, it is the dihedral group with 6 elements, isomorphic to S_3.)

8. Artin 7.2.5 on page 221.