Math 30810 Honors Algebra 3
Homework 12

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Due Wednesday, December 11

Do 5.

1. Let R be a ring. Show that $R[\![x]\!]^\times = R^\times + xR[\![x]\!]$.

2. Let R be a ring and I an ideal of $R[X]$. For a polynomial $P(X) \in R[X]$ let $\ell(P)$ be the leading coefficient of $P(X)$. Define $J = \{\ell(P) \mid P \in I\}$. Show that J is an ideal of R. (This is very useful.)

3. Consider the ring $R = \mathbb{Z}[\sqrt{-14}] = \{m + n\sqrt{-14} \mid m, n \in \mathbb{Z}\}$. Let $I = (3, 1 + \sqrt{-14})$. Show that $I^2 = (9, 7 + \sqrt{-14})$ and that $I^4 = (5 + 2\sqrt{-14})$ and thus that I^4 is a principal ideal. (One can, in fact, show that the fourth power of any ideal in this ring is principal, but this would be the topic of a graduate number theory course.)

4. Let R be a ring and p a prime number such that $p = 0$ in R. Show that $\phi(x) = x^p$ is a ring homomorphism $\phi : R \to R$. (The homomorphism ϕ is called the Frobenius homomorphism, which will be essential next semester.)

5. Artin 11.3.3 on page 354

6. Artin 11.3.4 on page 355.

7. Artin 11.3.9 on page 355.

8. Artin 11.3.10 on page 355.

9. Artin 11.4.4 on page 355.

10. Artin 11.6.7 on page 356.

11. Artin 11.M.7 on page 358.