Math 40520 Theory of Number
Homework 1

Due Wednesday, 8/26

Do 5 of the following.

1. Exercise 1.10 on page 19 in the textbook.
2. Exercise 1.12 on page 20 in the textbook.
3. Exercise 1.14 on page 20 in the textbook.
4. Solve $455x + 1235y = 65$ with $x, y \in \mathbb{Z}$.
5. Write 3.06015625 in base 20.
6. Here's three numbers in cuneiform (base 60):

   The first character is 1, the second is 10. The third number is written in base 60 in the form $x = \overline{a.bcd}$, where each of a, b, c, d is a "digit" between 0 and 59 (I added the fractional "."). Cuneiform digits are transparently constructed from 1s and 10s. What number is x approximating (I took it from a clay tablet from Wikipedia)?

7. Let $a_1, \ldots, a_n, b$ be non-zero integers. Show that the Diophantine equation

$$a_1x_1 + \cdots + a_nx_n = b$$

   has an integer solution if and only if $(a_1, \ldots, a_n) \mid b$.

8. Let $a, b \geq 1$ be two coprime integers. Show that every $n \geq (a-1)(b-1)$ can be written as $n = ax + by$ for $x, y \geq 0$. (This is the two stamp problem, sharp version. You may find a proof in Arthur Engel, "Problem Solving Strategies" (1991), problem 145 solved on page 156. You are welcome to read the solution there.)