

# Math 40520 Theory of Number

## Homework 2

Due Wednesday, 9/2

**Do 5.**

1. Show that the equation

$$x^2 + y^2 + z^2 = 20152015$$

has no integral solutions. [Hint: Try congruences modulo powers of 2.]

2. Show that the equation  $7x^2 - y^2 = 2^z$  has no integer solutions.  
3. Show that the equation  $x^2 - xy + y^2 = 2018$  has no integer solutions.  
4. Solve explicitly the equation  $2x + 3y + 5z = 7$  in the integers.  
5. Let  $p$  be a prime and consider the rational number

$$\frac{m}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{p-1}$$

If  $p > 2$  show that  $p \mid m$ . [Hint: consider the function  $f : \mathbb{Z}_p^\times \rightarrow \mathbb{Z}_p^\times$  defined by  $f(x) = x^{-1}$ .]

6. Write  $\frac{1}{7}$  as a decimal in base 5.  
7. Determine  $x \in \mathbb{Z}_{2020}$  such that

$$x^{97} \equiv 1351 \pmod{2020}.$$

8. How many digits does the 1000000th Fibonacci number have when written in base 7?