Math 40520 Theory of Number
Homework 5

Due Wednesday 9/23

Do 5.

1. Find the last 2 digits of $6^{6^{6^{6^6}}}$.
2. Determine $v_3(5^n - 1)$ as a function of $v_3(n)$. (Careful, $3 \nmid 5 - 1$ so this is similar to the $3 \nmid 2 - 1$ case from class, and the answer will have cases.)
3. What is $v_{11}(3^{146410} - 2^{146410})$?
4. What is $v_7(3^{5402250} - 2^{31513125})$? [Hint: This is not as hard as it looks. Rewrite the difference as a sum of two expressions of the form $a^n - b^n$.]
5. Show that $\pm 1, \pm 3, \dots, \pm 3^{2^{n-2}-1}$ are all distinct modulo 2^n. [Hint: Recall that the order of $3 \mod 2^n$ is 2^{n-2}. The harder part will be to show that $3^{2^{n-3}} \not\equiv -1 \pmod{2^n}$, when $n \geq 3$, but then you can use that $3^{2^{n-3}} \equiv 1 \pmod{2^{n-1}}$.]
6. Let a be a positive integer. Find the smallest positive integer k such that $2^{2020} \mid 2049^k - 1$. [Hint: Review how we computed the multiplicative order of $3 \mod 2^n$.]
7. What is the order of 5 modulo 2^{100}?
8. (Version of 2.32 on page 47) For each a between 1 and 100 compute the proportion of primes $100 < p < 10^6$ such that a is a primitive root mod p. Make a guess about the pattern. (This is a programming exercise, feel free to use Sage.)