Math 40520 Theory of Number
Homework 10

Due Wednesday, 11/4

Do 4.
For some of these questions you'll need a computer.

1-2 (Counts as 2 problems)

Consider the equation $x^2 + 11y^2 = 3$.

(a) Show that for all n it has solutions mod 2^n of the form $(0, y)$. [Hint: Recall that $x^2 \equiv u \pmod{2^n}$ has solutions for all n as long as $u \equiv 1 \pmod 8$.]

(b) Show that it has solutions mod 3^n of the form $(x, 1)$.

(c) Show that it has solutions mod 11^n of the form $(x, 0)$.

(d) If $p \notin \{2, 3, 11\}$ is a prime show that it has a solution (x_0, y_0) mod p and conclude that it must have a solution mod p^n either of the form (x, y_0) or (x_0, y).

(e) Use Hasse's theorem to conclude that the equation has solutions in $\mathbb{Q}$. (One such solution is $(1/2, 1/2)$.)

3. Textbook exercise 5.3 on page 121.

4. Textbook exercise 5.4 on page 121.

5. Textbook exercise 5.9 on page 121.

6. A fraction $\frac{a}{b} \approx 0.515287517$ has a and b with 4 digits. What are they?

7. A polynomial $P(X) \in \mathbb{Q}[X]$ is a product of 2 linear and 2 quadratic irreducible polynomials. Find these factors knowing only the approximation

$$P(X) \approx X^6 - 5.380846X^5 + 1.723134X^4 + 21.724378X^3 - 14.273383X^2 - 10.412687X + 1.997512.$$

8. Suppose $x > 0$ is a real number. Find positive integers $a_0, \ldots, a_n$ such that $\frac{7x+3}{5x+2} = [a_0; a_1, \ldots, a_d, x]$.