

Math 40520 Theory of Number

Homework 10

Due Wednesday, 11/11

Do 4.

For some of these questions you'll need a computer.

1. Suppose a real number x has the following continued fractions: $x = [4; 3, 2, 5, 1, a_5, a_6, a_7, \dots]$ where a_n are integers. Show that

$$\left| x - \frac{193}{45} \right| < \frac{1}{3735}.$$

2. Find a solution to Pell's equation $x^2 - 127y^2 = 1$.
3. Knowing that $x = 2175, y = 193$ is a solution to $x^2 - 127y^2 = 2$, can you use the previous exercise to find another solution? [Hint: Use that $N(x + y\sqrt{127}) = x^2 - 127y^2$ is a multiplicative function.]
4. (This is straightforward) Suppose $a_0, a_1, \dots \in (0, \infty)$ are reals and denote $c_n = [a_0; a_1, \dots, a_n]$. Recall the notation p_n, q_n from class.
 - (a) Show that (q_n) is nondecreasing.
 - (b) Conclude that $q_n \geq a_n + q_{n-2}$ for all n .
 - (c) Deduce that $q_n q_{n+1} \geq \left(\sum a_{n-2k} \right) \left(\sum a_{n+1-2k} \right)$.
 - (d) Finally, conclude that if $\sum a_n$ diverges then $q_n q_{n+1}$ diverges and therefore that (c_n) converges to some real number $[a_0; a_1, a_2, \dots]$.
5. Find the fraction $\frac{p}{q}$ closest to e subject to the condition that $q < 10^7$.
6. Textbook exercise 5.1 on page 121.
7. Textbook exercise 5.5 on page 121.