Math 30820 Honors Algebra 4
Homework 1

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Due Wednesday, 1/29/2020

Do 5.

1. Artin 14.2.3 (a) and from (b) the "only if" part on page 437.
2. Artin 14.2.4 on page 437.
3. Artin 14.4.3 on page 438.
4. Artin 14.7.9 on page 439.
5. Artin 14.8.2 on page 440. (Here the "corresponding linear operator" refers to multiplication by t.)
6. Suppose α is integral over a ring R, i.e., there exists a monic polynomial in $R[X]$ vanishing at α. Show that the R-module $R[\alpha]$ (defined last semester as $\{P(\alpha) \mid P \in R[X]\}$) is in fact the set $\{a_0 + a_1\alpha + \cdots + a_n\alpha^n \mid a_0, \dots, a_n \in R\}$ for some integer n.
7. Suppose R is a commutative ring with unit and M is an R-module. Define the annihilator of M in R as $\mathrm{Ann}_R(M) = \{r \in R \mid rm = 0, \forall m \in M\}$.
   (a) Show that $\mathrm{Ann}_R(M)$ is an ideal of R.
   (b) Show that if $I \subset \mathrm{Ann}_R(M)$ is an ideal of R then M is naturally an R/I-module.
   (c) What is $\mathrm{Ann}_R(R/I)$?
8. Let $M_1, \dots, M_n$ be R-modules, and let $N_1 \subset M_1, \dots, N_n \subset M_n$ be R-submodules. Show that

$$M_1 \oplus \ldots \oplus M_n / N_1 \oplus \ldots \oplus N_n \cong M_1/N_1 \oplus \ldots \oplus M_n/N_n.$$