Math 30820 Honors Algebra 4
Homework 7

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Due Wednesday, 3/18/2020

Do 5.

Throughout this problem set $\Phi_n(X)$ is the n-th cyclotomic polynomial.

1. Show that $\Phi_{2n}(X) = \Phi_n(-X)$ for any odd $n > 1$.

2. Let $a \in \mathbb{Z}$. Show that if p is an odd prime divisor of $\Phi_n(a)$ then either $p \mid n$ or $n \mid p - 1$.

3. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a field automorphism.

   (a) Show that $f|_{\mathbb{Q}} = \mathrm{id}_{\mathbb{Q}}$.

   (b) Show that if $x > 0$ then $f(x) > 0$ and conclude that f is increasing.

   (c) Show that if $|x - y| < \dfrac{1}{n}$ then $|f(x) - f(y)| < \dfrac{1}{n}$ and conclude that f is continuous.

   (d) Show that $f = \mathrm{id}_{\mathbb{R}}$.

4. Let $F = k(x)$ be the field of rational functions in the variable x with coefficients in some field k. Suppose $\phi : F \to F$ is a field automorphism such that $\phi|_k = \mathrm{id}|_k$. Show that there exists $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{GL}(2, k)$ such that $\phi(x) = \dfrac{ax + b}{cx + d}$ and conclude that $\mathrm{Aut}(k(t)/k) \cong \mathrm{PGL}_2(k)$.

5. Artin 15.7.5 on page 474.

6. Artin 15.7.8 on page 474.

7. Artin 15.7.12 on page 474.

8. Artin 15.7.14 on page 474.