

Math 30810 Honors Algebra 3

Homework 1

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Due Wednesday, September 8 in class

Do 5.

1. Artin 1.6 on page 32.
2. Artin 1.9 on page 32.
3. Artin 1.13 on page 32. [Hint: Do you know a good formula for $\frac{1}{1+x}$ where x is a real number with $|x| < 1$?]
4. Artin 5.2 on page 127.
5. If $A, B \in M_{n \times n}(\mathbb{R})$ we say that $A \sim_{\mathbb{R}} B$ if there exists an invertible matrix $S \in M_{n \times n}(\mathbb{R})$ such that $A = SBS^{-1}$. Similarly, we say $A \sim_{\mathbb{C}} B$ if there exists an invertible matrix $S \in M_{n \times n}(\mathbb{C})$ such that $A = SBS^{-1}$. Show that if $A \sim_{\mathbb{C}} B$ then $A \sim_{\mathbb{R}} B$. [Hint: Use the real and imaginary parts.]
6. Suppose $A \in M_2(\mathbb{C})$ is a matrix such that $\{A\}$ together with matrix multiplication forms a group with 1 element.
 - (a) Show that the eigenvalues of A are either 0 or 1. If both are 0, show that $A = 0_2$; if both are 1, show that $A = I_2$; if one is 0 and one is 1 show that $\text{tr } A = 1$ and $\det A = 0$. [Hint: Use Jordan forms.]
 - (b) Show that any matrix $A \in M_2(\mathbb{C})$ with trace 1 and determinant 0 satisfies the property that $\{A\}$ together with matrix multiplication forms a group.. [Hint: Cayley-Hamilton.]
7. Let $X_n = \{1, 2, \dots, n\}$.
 - (a) Show that there exist n^{n^2} composition laws on X_n .
 - (b) Of the 3^9 composition laws on X_3 how many:
 - i. are commutative?
 - ii. have a unit?
 - iii. have a unit and are associative?
 - iv. form a group?
 - v. form a commutative group?