Math 30810 Honors Algebra 3
Homework 3

Andrei Jorza

Due Wednesday, September 22

Do 5 questions.

1. Artin 2.4.3 or 2.6.4
2. Artin 2.5.2
3. Artin 2.6.2
4. Artin 2.6.8 or 2.6.9
5. Artin 2.7.5
6. Let G be a group with subgroups H and K. Show that $H \cup K$ is a group if and only if one of H and K contains the other.
7. Recall from class that if S is the set of smooth functions defined on some open neighborhood of 0 in $\mathbb{R}$ then we have an equivalence relation $\sim$ that identifies two functions that agree on some open neighborhood of 0. Define a natural multiplication function on $S/\sim$. For this multiplication, determine the identity and the invertible elements.
8. Suppose $f : G \to \mathrm{GL}_n(\mathbb{R})$ is a group homomorphism with the property that for every $x \in G$, $f(x)$ has integer entries. Show that $\mathrm{Im}\, f$, which a priori is a subgroup of $\mathrm{GL}_2(\mathbb{R})$, is in fact a subgroup of $\mathrm{GL}_n(\mathbb{Z})$.