Math 30810 Honors Algebra 3
Homework 8

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Due in class Wednesday, November 17

Do 5.

1. Show that (ij) and $(12\dots n)$ generate S_n if and only if $(i-j,n)=1$.
2. Artin 7.2.15
3. Artin 7.2.17
4. Artin 7.3.3
5. Artin 7.5.7
6. Artin 7.5.12
7. Let $n \geq 5$.
   (a) Suppose $\sigma = c_1c_2\cdots c_r \in S_n$ is a product of disjoint cycles with c_1 of length ≥ 3. Find $\tau \in S_n$ such that $\tau\sigma\tau^{-1}\sigma^{-1}$ is a 3-cycle.
   (b) Suppose $\sigma = c_1c_2\cdots c_r \in S_n$ is a product of disjoint transpositions with $r \geq 1$. Find $\tau \in S_n$ such that $\tau\sigma\tau^{-1}\sigma^{-1}$ is a product of 2 disjoint transpositions.
   (c) Suppose $\sigma = c_1c_2$ is a product of disjoint transpositions. Find $\tau \in S_n$ such that $\tau\sigma\tau^{-1}\sigma^{-1}$ is a 3-cycle.
   (d) Let $N \lhd S_n$. Show that if $N \neq \{1\}$ then $N \supset A_n$, and therefore that $N = A_n$ or $N = S_n$. [Hint: Recall from class that 3-cycles generate A_n.]
8. Enumerate the orbits of $\mathrm{SL}_2(\mathbb{Z})$ on $\mathbb{Z}^2 - \{0\}$.
9. Suppose G acts on X. Show that for each $g \in G$, $x \in X$, $\mathrm{Stab}_G(gx) = g\,\mathrm{Stab}_G(x)g^{-1}$.
10. Let G be a group, $H < G$ and $N \lhd G$ such that $H \cap N = \{1\}$.
    (a) Show that the multiplication map $N \times H \to NH$ is a bijection of sets.
    (b) Find a homomorphism $\phi : H \to \mathrm{Aut}(N)$ such that the above multiplication map gives an isomorphism of groups $N \rtimes_\phi H \cong NH$.