Math 30810 Honors Algebra 3
Homework 9

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Due Wednesday, December 1

Do 5.

1. Artin 6.M.7 on page 194.
2. Artin 7.2.5 on page 221.
3. Artin 7.7.3 on page 222.
4. Artin 7.7.4 on page 222.
5. Artin 7.7.8 on page 222.
6. Artin 7.7.9 on page 222.
7. Artin 7.7.10 on page 222.
8. Artin 7.8.3 on page 222.
9. (Counts as 2 problems) Let G be a finite group and H a subgroup of G. Denote by $S_{G/H}$ the group of permutations of the finite set G/H. If G/H has k elements then $S_{G/H} \cong S_k$, the group operation on both sides being composition of permutations.
   (a) Show that if $g \in G$ then the map $f_g : G/H \to G/H$ defined by $f(xH) = gxH$ is a bijection, in other words $f_g \in S_{G/H}$.
   (b) Show that the map $\Phi : G \to S_{G/H}$ given by $\Phi(g) = f_g$ is a group homomorphism with $\ker \Phi \subset H$.
   (c) Suppose the index $[G : H] = p$ is the least prime divisor of the order $|G|$. Show that $|G/\ker \Phi| = p$ and deduce that H is normal in G. (This is a generalization of a previous homework problem that stated that index 2 subgroups are normal. Indeed 2 is the least prime divisor of every even order.) [Hint: what is the cardinality of $S_{G/H}$?]