Math 40520 Theory of Numbers
Homework 1

Due Wednesday, in class, 9/7

Do 5 of the following.

1. Exercise 1.10 on page 19 in the textbook.
2. Exercise 1.12 on page 20 in the textbook.
3. Exercise 1.14 on page 20 in the textbook.
4. Solve $455x + 1235y = 65$ with $x, y \in \mathbb{Z}$.
5. Write 3.06015625 in base 20.
6. Here's three numbers in cuneiform (base 60):

   The first character is 1, the second is 10. The third number is written in base 60 in the form $x = \overline{a.bcd}$, where each of a, b, c, d is a "digit" between 0 and 59 (I added the fractional "."). Cuneiform digits are transparently constructed from 1s and 10s. What number is x approximating (I took it from a clay tablet from Wikipedia)?

7. Suppose a and b are positive integers and p is a prime number such that $a^p = b! + p$. Show that $b < 2p$. [Hint: look at how p divides both sides.]
8. (This is not a hard exercise, even if it looks very long.) In this exercise you will multiply two positive integers using only doubling, halving and additions. Suppose m and n are two positive integers. Put m and n on the same row in a table with two columns. You will iterate the following operation. Taking the last row of the column, multiply by 2 the left entry and divide by 2 the right entry and put the new values on the next row, forgetting about decimals. When the right row becomes 0, stop the iteration. Eliminate from the column every row in which the right entry is even, then add all the remaining left entries. This sum will then be the product $m \cdot n$. For example

$x \times 2$	$\lfloor x/2 \rfloor$
23	25
~~46~~	~~12~~
~~92~~	~~6~~
184	3
368	1
~~736~~	~~0~~

yield $23 \cdot 25 = 575 = 368 + 184 + 23$.

(a) Write $m = \overline{m_1 m_2 \cdots m_k}_{(2)}$ and $n = \overline{n_1 n_2 \ldots n_k}_{(2)}$ in base 2. Show that the table, all entries written in base 2, is

$x \times 2$	$\lfloor x/2 \rfloor$
$\overline{m_1 m_2 \cdots m_k}$	$\overline{n_1 n_2 \ldots n_k}$
$\overline{m_1 m_2 \cdots m_k 0}$	$\overline{n_1 n_2 \ldots n_{k-1}}$
$\overline{m_1 m_2 \cdots m_k 00}$	$\overline{n_1 n_2 \ldots n_{k-2}}$
$\vdots$	$\vdots$
$\overline{m_1 m_2 \cdots m_k \underbrace{00\ldots0}_{k-1}}$	$\overline{n_1}$
$\overline{m_1 m_2 \cdots m_k \underbrace{00\ldots0}_{k}}$	0

(b) Show that the algorithm is correct. [Hint: Write out multiplication in base 2.]