Math 40520 Theory of Number
Homework 2

Due Wednesday, in class, 9/14

Do 5.

1. Exercise 2.4 in the textbook.

2. Exercise 2.5 in the textbook.

3. Solve explicitly the equation $2x+3y+5z = 7$ in the integers. [Hint: You know how to solve $2x+3y = N$ for all N.]

4. Show that the equation
$$x^2 + y^2 + z^2 = 20152015$$
has no integral solutions. [Hint: Try congruences modulo powers of 2.]

5. Show that the equation $7x^2 - y^2 = 2^z$ has no non-negative integer solutions. [The negatives are easy: z has to be non-negative, or else the LHS is not integral.]

6. Show that the equation $x^2 - xy + y^2 = 2021$ has no integer solutions. [A slightly harder alternative: show that the equation $x^2 - xy + y^2 = 2022$ has no integer solutions.]

7. Write $\frac{1}{7}$ as a decimal in base 5.

8. Consider the Lucas sequence (L_n) defined using the same recurrence as the Fibonacci sequence $L_{n+2} = L_{n+1} + L_n$, but with initial terms $L_0 = 2$, $L_1 = 1$. How many digits does the 1000000th Lucas number have when written in base 7?