Math 40520 Theory of Number
Homework 5

Due Wednesday 10/05

Do 5.

1. Exercise 4.7 on page 90.
2. Exercise 4.8 on page 91.
3. How many quadratic residues modulo 2027 are there between 1 and 10? Explain your reasoning.
4. Determine all residues $x \pmod{140}$ such that $x^4 - x - 84 \equiv 0 \pmod{140}$.
5. Find all $x \pmod{1999}$ such that $x^3 \equiv 2 \pmod{1999}$.
6. Find all $x \pmod{1997}$ such that $x^5 \equiv 17 \pmod{1997}$.
7. Show that the polynomial $(X^2 - 2)(X^2 - 3)(X^2 - 6)$ has a root in $\mathbb{Z}_p$ modulo every prime p.
8. (Version of 2.32 on page 47) For each a between 1 and 100 compute the proportion of primes $100 < p < 10^6$ such that a is a primitive root mod p. Make a guess about the pattern; feel free to review the part of lecture 14 that I didn't do in class. (This is a programming exercise, feel free to use Sage.)