

Math 40520 Theory of Number

Homework 6

Due Wednesday, 11/2, in class

Do 5.

1. Exercise 4.9 on page 91.
2. Compute, by hand (you can use a calculator for multiplying integers), $\binom{667}{2027}$.
3. Compute $\binom{194871}{1610} \pmod{385}$. [Hint: Use our theorem for binomial coefficients modulo primes (Lucas' theorem) and the Chinese Remainder Theorem.]
4. Determine $v_7(\varphi(200!))$.
5. Show that the coefficient of x^n in the Taylor series around 0 of $\frac{1}{\sqrt{1-x}}$ is $\frac{1}{4^n} \binom{2n}{n}$.
6. What is the 3-valuation of the coefficient of x^{2023} in the Taylor series around 0 of $\arcsin x$?
7. Show that $(100!)^{50}$ divides $\prod_{k=1}^{100} k^k$.
8. Suppose m and n are integers such that

$$m! = (2^n - 1)(2^n - 2)(2^n - 2^2) \cdots (2^n - 2^{n-1}).$$

Show that $m > \binom{n}{2}$.