Hedging

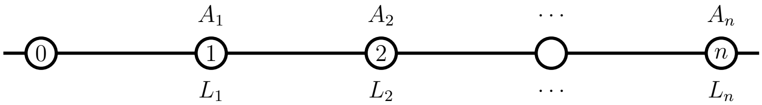
Hedging is the process by which a portfolio is adjusted to mitigate risk.

For instance, suppose you think BP stock will fall and would like to short sell it. What might go wrong?

- The stock market might go up.
- The oil industry might go up.

Hedging against IR

Immunization is the process of hedging against interest rate fluctuations.



- Receive $A_1, \ldots, A_n$
- Pay $L_1, \ldots, L_n$
- PV of portfolio is

$$\mathrm{PV}_i = (A_1 - L_1)\nu_i + (A_2 - L_2)\nu_i^2 + \cdots + (A_n - L_n)\nu_i^n.$$

Goal for portfolio: $P(i) = \mathrm{PV}_i \geq 0$.

Goal for chapter 7:

- How does $P(i)$ change with i?
- Can we set up the portfolio so that $P(i) \geq 0$ as i varies?

Exercise

A company has the following assetts and liabilities:

1. Asset: 5-year annuity paying 1m immediately.
2. Asset: One 10m income at the end of the 2nd year.
3. Liability: Needs to pay 3-year annuity paying 5m starting with the end of the 1st year.

Immunization

Suppose you have a portfolio with various assets (i.e., sources of income) and various liabilities (i.e., payments you need to effect).

Goal: "Assets should cover liabilities"

- For each "cost of capital interest rate i" you compute the present value $P(i)$ of all assets and liabilities. The portfolio is **fully immunized** if the smallest value of $P(i)$ is $\min P(i) \geq 0$.
- To **immunize** the portfolio means to add/change assets in such a way that it becomes (fully) immunized.
- Terminology: the portfolio is **fully immunized at** $i = i_0$ means that $\min P(i)$ is attained at $i = i_0$ and $\min P(i) = P(i_0) \geq 0$.

Around Exercise 7.2.6

Example

A company has a \$1m liability payable in 12 years, and income of \$15k yearly for 12 years, starting now. To cover the liability the company adds an asset that pays A at time t.

1. Employee 1 says this works with $A = \$500\text{k}$ at time $t = 10$ years.
2. Employee 2 says this works with $A = \$1\text{m}$ at time $t = 15$.
3. Employee 3 says that the portfolio can be immunized at $i = 10\%$.

Around Exercise 7.2.6

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3. Employee 3 says that the portfolio can be immunized at $i = 10\%$.

Let's compute the present value (in thousands)

$$P(i) = 15\ddot{a}_{\overline{12}|i} - 1000\nu_i^{12} + A\nu_i^t$$

Exercise 7.2.6, Employee 1

Employee 1 says this works with $A = \$500$k at time $t = 10$ years $P(i) = 15\ddot{a}_{\overline{12}|i} + 500\nu_i^{10} - 1000\nu_i^{12}$.

Exercise 7.2.6, Employee 1

Employee 1 says this works with $A = \$500$k at time $t = 10$ years $P(i) = 15\ddot{a}_{\overline{12}|i} + 500\nu_i^{10} - 1000\nu_i^{12}$.

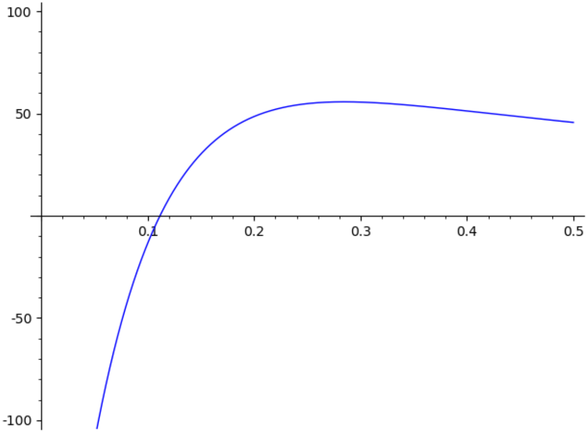


Figure: Plot of $P(i)$

Lecture 38 April 26, 2023

Immunization

Assets should cover liabilities:

$$P(i) = \mathrm{NPV}_i = \sum (A_t - L_t)\nu_i^t \geq 0.$$

- **Fully immunized** means $\min P(i) \geq 0$.
- **Fully immunized at $i = i_0$** means that $\min P(i)$ is attained at $i = i_0$ and $\min P(i) = P(i_0) \geq 0$.

Limiting cases:

i	ν_i	$P(i)$
0	1	$\sum (A_t - L_t)$
∞	0	$A_0 - L_0$

Around Exercise 7.2.6

Example

A company has a \$1m liability payable in 12 years, and income of \$15k yearly for 12 years, starting now. To cover the liability the company adds an asset that pays A at time t.
The present value (in thousands)

$$P(i) = 15\ddot{a}_{\overline{12}|i} - 1000\nu_i^{12} + A\nu_i^t$$

Exercise 7.2.6, Employee 2

Employee 2 says this works with $A = \$1$m at time $t = 15$ years $P(i) = 15\ddot{a}_{\overline{12}|i} - 1000\nu_i^{12} + 1000\nu_i^{15}$.

Exercise 7.2.6, Employee 2

Employee 2 says this works with $A = \$1\text{m}$ at time $t = 15$ years $P(i) = 15\ddot{a}_{\overline{12}|i} - 1000\nu_i^{12} + 1000\nu_i^{15}$.

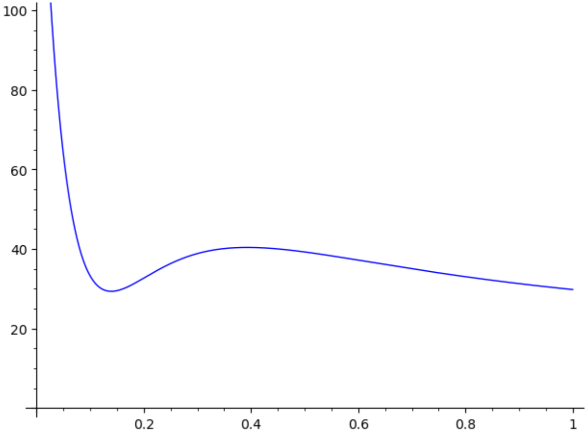


Figure: Plot of $P(i)$

Exercise 7.2.6, Employee 2 computations

The previous plot is for i between 0 and 100%, but interest rates can, a priori, be arbitrarily high. Let's check that, in fact, the portfolio is immunized.
How do we find $\min P(i)$:

$$\min P(i) = \min 15(1+(1+i)^{-1}+(1+i)^{-2}+\cdots+(1+i)^{-11})-$$

$$-1000(1+i)^{-12}+1000(1+i)^{-15}$$

Exercise 7.2.6, Employee 2 computations

Critical points are $i_1 = 13.94\%$ and $i_2 = 39.30\%$.

$P(i_1)$	29.32
$P(i_2)$	40.36
$P(0)$	180
$P(\infty)$	15

So Employee 2 was right, the portfolio is fully immunized:

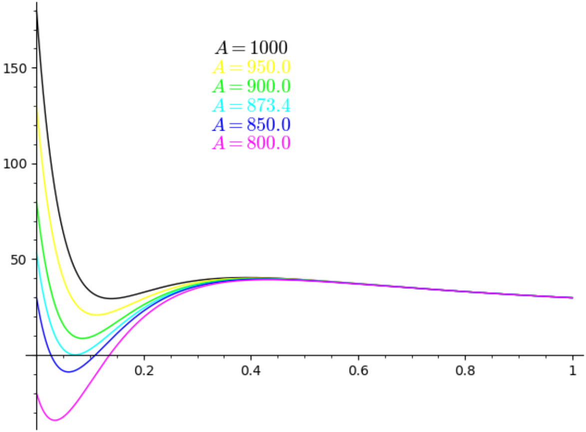
$$P(i) \geq 15 \qquad \text{for all values of } i$$

In fact, there are too many assets compared to the liability.

Exercise 7.2.6, Employee 2 Making do with less

Theorem

Suppose you have a portfolio $\mathcal{P}$ that is fully immunized. Then you can decrease some assets so that the portfolio becomes **exactly immunized**, i.e., $\min P(i) = 0$.



Exercise 7.2.6, Employee 3

Employee 3 tells us that the portfolio can be fully immunized at $i = 10\%$, which means that for some values of A and t, $P(i)$ attains its minimum exactly when $i = 10\%$.
The **Theorem** also tells us that we can **exactly immunize**, i.e., this minimum value is 0.

$$P(10\%) = 0$$
$$\min P(i) = P(10\%)$$

Exercise 7.2.6, Employee 3

Employee 3 tells us that the portfolio can be fully immunized at $i = 10\%$, which means that for some values of A and t, $P(i)$ attains its minimum exactly when $i = 10\%$.
The **Theorem** also tells us that we can **exactly immunize**, i.e., this minimum value is 0.

$$P(10\%) = 0$$
$$\min P(i) = P(10\%)$$

What does this mean? It means that $i = 10\%$ is also a **critical value** so

$$P(10\%) = 0$$
$$P'(10\%) = 0$$

Exercise 7.2.6, Employee 3 computations

$$P(10\%) = -206.2049 + \frac{A}{1.1^t} = 0$$
$$P'(10\%) = 3027.4542 - \frac{At}{1.1^{t+1}} = 0$$

Exercise 7.2.6, Employee 3 computations

$$P(10\%) = -206.2049 + \frac{A}{1.1^t} = 0$$
$$P'(10\%) = 3027.4542 - \frac{At}{1.1^{t+1}} = 0$$

Get

$$t = 16.15$$
$$A = 961.149$$

Redington Immunization: A first example

Example

A company pays \$243k now, will receive \$1.35m in 1 year and \$1m in 3 years, and must pay \$2.1m in 2 years. Is this portfolio immunized?

Redington Immunization: A first example

Example

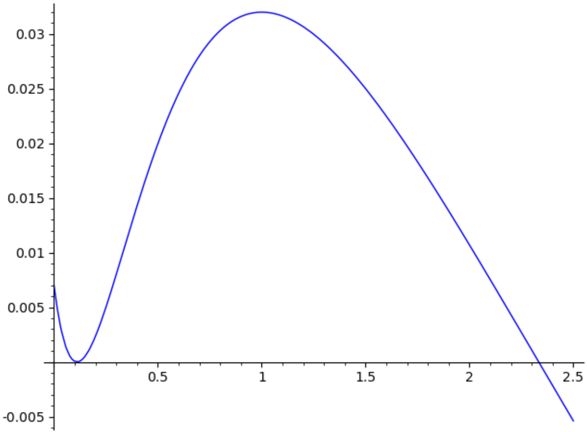
A company pays \$243k now, will receive \$1.35m in 1 year and \$1m in 3 years, and must pay \$2.1m in 2 years. Is this portfolio immunized?

Let's compute the PV (in millions):

$$\begin{aligned} P(i) &= -0.243 + 1.35\nu_i - 2.1\nu_i^2 + \nu_i^3 \\ &= -0.243 + \frac{1.35}{1+i} - \frac{2.1}{(1+i)^2} + \frac{1}{(1+i)^3} \end{aligned}$$

Redington Immunization: A first example

Is this portfolio fully immunized? In other words, is $P(i) \geq 0$?



(Roots are 11.11% and 233.33%)

Redington Immunization: Definition

Definition

A portfolio is **Redington Immunized** at $i = i_0$ if

- $P(i_0) = 0$ (you can always assume this) and
- $P(i) \geq 0$ if i varies "slightly around i_0".

Redington Immunization: Definition

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A portfolio is **Redington Immunized** at $i = i_0$ if

- $P(i_0) = 0$ (you can always assume this) and
- $P(i) \geq 0$ if i varies "slightly around i_0".

In other words, if $P(i)$ has a local minimum at $i = i_0$.

Theorem (Second Derivative Test)

To test if $P(i)$ has a local minimum at $i = i_0$ check:

- $P'(i_0) = 0$ *(critical point)*
- $P''(i_0) > 0$ *(convex up)*

Exercise 7.2.7 Redux

Example

A portfolio has liability payments of 100 in 2,4, and 6 years and asset incomes of a in 1 year and b in 5 years. Can this portfolio be Redington Immunized at $i = 10\%$?

Exercise 7.2.7 Redux

Example

A portfolio has liability payments of 100 in 2,4, and 6 years and asset incomes of a in 1 year and b in 5 years. Can this portfolio be Redington Immunized at $i = 10\%$?

The present values are $P_A(i) = a(1+i)^{-1} + b(1+i)^{-5}$ and $P_L(i) = 100(1+i)^{-2} + 100(1+i)^{-4} + 100(1+i)^{-6}$ for a portfolio present value of $P(i) = P_A(i) - P_L(i)$.
What does it mean to be Redington Immunized at $i = 10\%$?

- $P_A(10\%) = P_L(10\%)$ (assets exactly cover liabilities)
- $P_A'(10\%) = P_L'(10\%)$ (critical point at $i = 10\%$)
- $P_A''(10\%) > P_L''(10\%)$ (second derivative test for local minimum at $i = 10\%$)

Exercise 7.2.7 Computations

$P_A(10\%) = P_L(10\%)$ becomes

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$P_A(10\%) = P_L(10\%)$ becomes

$$\frac{a}{1.1} + \frac{b}{1.1^5} = \frac{100}{1.1^2} + \frac{100}{1.1^4} + \frac{100}{1.1^6} = 207.39$$

$P'_A(10\%) = P'_L(10\%)$ becomes

Exercise 7.2.7 Computations

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$$\frac{a}{1.1} + \frac{b}{1.1^5} = \frac{100}{1.1^2} + \frac{100}{1.1^4} + \frac{100}{1.1^6} = 207.39$$

$P'_A(10\%) = P'_L(10\%)$ becomes

$$-\frac{a}{1.1^2} - \frac{5b}{1.1^6} = -\frac{200}{1.1^3} - \frac{400}{1.1^5} - \frac{600}{1.1^7} = -777.18$$

Exercise 7.2.7 Computations

$P_A(10\%) = P_L(10\%)$ becomes

$$\frac{a}{1.1} + \frac{b}{1.1^5} = \frac{100}{1.1^2} + \frac{100}{1.1^4} + \frac{100}{1.1^6} = 207.39$$

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Solve and get $a = 71.44$ and $b = 229.41$. So does this satisfy Redington Immunization? I.e., is $P''_A(10\%) > P''_L(10\%)$.

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$$P''_A(10\%) = \underbrace{\frac{2a}{1.1^4} + \frac{5 \cdot 6b}{1.1^7}}_{4403} > P''_L(10\%) = \underbrace{\frac{600}{1.1^4} + \frac{2000}{1.1^6} + \frac{4200}{1.1^8}}_{3225}$$

Cashflow duration: a reinterpretation of $P'(i)$

In verifying exact immunization at an interest rate i we must check

$$P_A(i) = P_L(i)$$
$$P'_A(i) = P'_L(i)$$

Let's focus on $P_A(i)$, assets with incomes A_t at times t for a number of times t:

1. $P_A(i) = \sum A_t \nu_i^t = \sum A_t (1+i)^{-t}$.

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3. $P''_A(i) = \sum t(t+1) A_t (1+i)^{-t-2}$.

Lecture 39 April 28, 2023

Redington Immunization

Want $P(i) = P_A(i) - P_L(i)$ to have a local minimum at $i = i_0$.

$P(i_0) = 0$	Solve for assets	$P_A(i_0) = P_L(i_0)$
$P'(i_0) = 0$		$P'_A(i_0) = P'_L(i_0)$
$P''(i_0) > 0$	Independent check	$P''_A(i_0) > P''_L(i_0)$

1. $P_A(i) = \sum A_t \nu_i^t$.
2. $P'_A(i) = \sum -tA_t \nu^{t+1}$.
3. $P''_A(i) = \sum t(t+1)A_t \nu^{t+2}$.

Cashflow duration: a reinterpretation of $P'(i)$

$$P'_A(i) = -\frac{\sum tA_t\nu_i^t}{1+i}$$

Definition

Macaulay had the idea to reinterpret the sum above in terms of a **Macaulay duration**. The idea is that asset A_t will occur at time t so its **duration** (time until asset is cashed) is t. The way to combine durations of various assets is to compute a weighted average of the assets' duration using as weight the PV of the asset as a percentage of the PV of the whole portfolio.

$$D_A(i) = \frac{\sum tA_t\nu_i^t}{\sum A_t\nu_i^t} \qquad D_A(i) = -\frac{P'_A(i)(1+i)}{P_A(i)}$$

Cashflow duration: a simple example

Example

You get 3 at time 2 and 5 at time 3. What is the Macaulay duration at $i = 5\%$?

Cashflow duration: a simple example

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You get 3 at time 2 and 5 at time 3. What is the Macaulay duration at $i = 5\%$?

The PV's are $3\nu_i^2$ and $5\nu_i^3$ so the duration is

$$D = \frac{2 \cdot 3\nu_i^2 + 3 \cdot 5\nu_i^3}{3\nu_i^2 + 5\nu_i^3} = 2.49.$$

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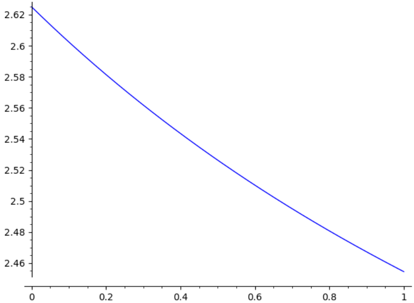

Cashflow duration: Bonds example

Example

What is the Macaulay duration of a bond with F, r, n at the yield rate i?

$$\begin{aligned} P(i) &= Fra_{\overline{n}|i} + F\nu_i^n = F + F(r-i)a_{\overline{n}|i} \\ &= Fr(\nu_i + \nu_i^2 + \cdots + \nu_i^n) + F\nu_i^n \end{aligned}$$

Cashflow duration: Bonds example

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The Macaulay duration is then

$$\begin{aligned} D &= \frac{Fr(\nu_i + 2\nu_i^2 + \cdots + n\nu_i^n) + nF\nu_i^n}{Fr(\nu_i + \nu_i^2 + \cdots + \nu_i^n) + F\nu_i^n} \\ &= \frac{Fr(Ia)_{\overline{n}|i} + nF\nu_i^n}{Fra_{\overline{n}|i} + F\nu_i^n} = \frac{r(Ia)_{\overline{n}|i} + n\nu_i^n}{ra_{\overline{n}|i} + \nu_i^n} \end{aligned}$$

(Remember $a_{\overline{n}|i} = (1-\nu^n)/i$ and $(Ia)_{\overline{n}|i} = (\ddot{a}_{\overline{n}|i} - n\nu^n)/i$.)

Mixing durations

Suppose you have a collection of portfolios for which you know $P_1(i), \ldots, P_n(i)$ and their durations $D_1(i), \ldots, D_n(i)$. What is the duration of the entire collection?

1. $P(i) = P_1(i) + \cdots + P_n(i)$

Example

At cost of capital i, two bonds have PV of 100 and 200 and durations 20 and 30. What is the duration of the two bonds together?

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2. $D(i) = D_1(i) \cdot \dfrac{P_1(i)}{P(i)} + \cdots + D_n(i) \cdot \dfrac{P_n(i)}{P(i)}$

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Example

At cost of capital i, two bonds have PV of 100 and 200 and durations 20 and 30. What is the duration of the two bonds together?

$$D = 20 \cdot \frac{100}{300} + 30 \cdot \frac{200}{300} = 26\frac{2}{3}.$$

Cashflow duration and Immunization

Remember that to immunize a portfolio at i means to (first) check

$$P_A(i) = P_L(i) \quad \& \quad P'_A(i) = P'_L(i)$$

$$P_A(i) = P_L(i) \quad \& \quad D_A(i) = D_L(i)$$

Remark

For assets to exactly cover liabilities you must have the two conditions:

1. *Assets and liabilities have the same PV.*

Cashflow duration and Immunization

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Remark

For assets to exactly cover liabilities you must have the two conditions:

1. *Assets and liabilities have the same PV.*
2. *Assets and liabilities have the same Macaulay duration.*
3. *(Alternatively, assets and liabilities have the same modified duration.)*

Cashflow duration and Redington Immunization

To immunize we need assets and liabilities to have the same PV and the same duration. This is not enough, and we also need

$$P''_A(i) > P''_L(i)$$

Definition

The **convexity** of a series of cashflows is $\dfrac{P''(i)}{P(i)}$.

Redington Immunization:

1. $P_A(i) = P_L(i)$.
2. $D_A(i) = D_L(i)$.
3. $\text{Convexity}_A(i) > \text{Convexity}_L(i)$.

Lecture 40 May 1, 2023

Redington Immunization

Want $P(i) = P_A(i) - P_L(i)$ to have a local minimum at $i = i_0$.

$$
\begin{array}{l|l|l}
P_A(i_0) = P_L(i_0) & P_A(i_0) = P_L(i_0) & \sum A_t \nu^t = \sum L_t \nu^t \\
P'_A(i_0) = P'_L(i_0) & D_A(i_0) = D_L(i_0) & \sum t A_t \nu^t = \sum t L_t \nu^t \\
P''_A(i_0) > P''_L(i_0) & \mathrm{Cv}_A(i_0) > \mathrm{Cv}_L(i_0) & \sum t^2 A_t \nu^t > \sum t^2 L_t \nu^t
\end{array}
$$

Duration and Redington Immunization: Example

Example

Assets a at 1, b at 7.

Liabilities 200 at 0, 140 at 3, 189 at 4.

Redington Immunize at $i = 5\%$.

1. Match PV: $a\nu + b\nu^7 = 200 + 140\nu^3 + 189\nu^4$.

Duration and Redington Immunization: Example

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Assets a at 1, b at 7.

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Redington Immunize at $i = 5\%$.

1. Match PV: $a\nu + b\nu^7 = 200 + 140\nu^3 + 189\nu^4$.
2. Match D: $a\nu + 7b\nu^7 = 3 \cdot 140\nu^3 + 4 \cdot 189\nu^4$.

Duration and Redington Immunization: Example

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3. Solutions are: $a = 411.289$ and $b = 119.216$.

Duration and Redington Immunization: Example

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Assets a at 1, b at 7.

Liabilities 200 at 0, 140 at 3, 189 at 4.

Redington Immunize at $i = 5\%$.

1. Match PV: $a\nu + b\nu^7 = 200 + 140\nu^3 + 189\nu^4$.
2. Match D: $a\nu + 7b\nu^7 = 3 \cdot 140\nu^3 + 4 \cdot 189\nu^4$.
3. Solutions are: $a = 411.289$ and $b = 119.216$.
4. Convexity check? $\underbrace{a\nu + 7^2 b\nu^7}_{4543} > \underbrace{3^2 \cdot 140\nu^3 + 4^2 \cdot 189\nu^4}_{3576}$?

   Yes!

Exercise

7.2.12.a

*7.2.12 It is assumed that the term structure of interest rates is flat at $j = .08$ per year. Suppose that a company has liabilities consisting of 10 annual payments of 1000 each starting in one year. You are given:

$$\sum_{k=1}^{10} v_{.08}^k = 6.7101, \quad \sum_{k=1}^{10} k v_{.08}^k = 32.6869, \quad \sum_{k=1}^{10} k^2 v_{.08}^k = 212.9687$$

(a) The company wishes to invest in assets in order to immunize the liabilities against small changes in j. The assets will consist of some cash now at time 0 and a zero coupon bond maturing at time 10. The present value and duration of the assets must match the present value and duration of the liabilities. Find how much of the asset portfolio should be in cash (nearest \$1).

(3441, 7057)

Other exercises

You are given the term structure $r_1 = 5\%$, $r_2 = 7\%$, $r_3 = 10\%$, $r_n = 13\%$ for $n \geq 4$. Assume that future interest rates are the forward interest rates given by this term structure.

An investor has 1m to invest. There are three options:

- Lend 1m at a constant interest rate i to be repaid in 5 level annual payments starting one year from now, and reinvest each payment at the market interest rate.
- Buy a 5-year 0-coupon bond.
- Buy a 5-year 6%-coupon bond with annual coupons.

The first two options will result in the same return on investment after 5 years. How much money does the investor have in the first option immediately after the 3rd payment? What is the yield to maturity of the third option?