

Math 162b Syllabus

p -adic Galois Representations

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1 Local Class Field Theory

1.1 Local fields

1.1.1 Hensel's lemma

Hensel's lemma and the Teichmüller character.

1.1.2 Krasner's lemma

Galois groups of local fields.

1.2 Newton polygons

Newton polygons and decomposability.

1.3 Ramification of local fields

1.3.1 Ramification

Basics of ramification theory, and interaction with Newton polygons.

1.3.2 Ramification filtrations

Lower filtration, ϕ , upper filtration, Hasse-Arf.

1.3.3 Different

Different and ramification.

1.4 Main results of local class field theory

Galois groups and ramification filtrations.

1.5 Galois cohomology

1.5.1 Continuous cohomology

Definition.

1.5.2 Inflation-restriction sequence

Definition.

2 \mathbb{C}_p -representations

2.1 The field \mathbb{C}_p

The field, why it's algebraically closed.

2.2 Ax-Sen-Tate and Galois invariants

2.2.1 Algebraic approximations

Lemma on approximations.

2.2.2 Galois invariants

Galois invariants of \mathbb{C}_p .

2.3 Ramification estimates and Tate periods

2.3.1 Ramification in cyclotomic extensions

Computation of ramification.

2.3.2 Almost etaleness

Traces and almost etaleness.

2.3.3 Traces

Tate generalized traces.

2.3.4 Tate periods

Cohomology of Tate twists of \mathbb{C}_p .

2.4 Hodge-Tate-Sen theory

2.4.1 Hodge-Tate representations

Definition and semisimplicity.

2.4.2 Galois descent

The descent procedure.

2.4.3 Decompletion

Decompletion procedure.

2.4.4 Sen's operator

Sen's operator, Hodge-Tate-Sen weights and Hodge-Tate representations.

3 Admissible Representations

3.1 B_{HT}

Hodge-Tate and graded vector spaces.

3.2 Regular rings

Definition.

3.3 Admissible representations

Definition and properties.

4 de Rham, Crystalline, Semistable

4.1 p -adic Analysis

4.1.1 p -adic Banach spaces

Banach spaces, Fréchet spaces.

4.1.2 Power series and log

Newton polygons for power series and log.

4.2 Witt vectors

4.2.1 The case of \mathbb{F}_p

Teichmüller lifts

4.2.2 Basics of Witt vectors

Main results

4.2.3 Perfections

R

4.3 B_{dR} and de Rham representations

4.3.1 Definition of B_{dR}

Construction

4.3.2 Cohomology of B_{dR}

Galois cohomology

4.3.3 de Rham representations

de Rham admissibility and filtered vector spaces.

4.4 B_{cris} and crystalline representations

4.4.1 Definition and Frobenius

A_{cris} , Frobenius, filtrations.

4.4.2 Crystalline representations

Crystalline representations and filtered modules with Frobenius.

4.4.3 Formal groups and crystallinity

Formal groups, differentials, de Rham cohomology and crystallinity.

4.5 B_{st} and semistable representations

4.5.1 Definition and monodromy

B_{st} , uniformizers and B_{dR} .

4.5.2 Semistable representations

Semistable representations and filtered modules with Frobenius and monodromy.

4.5.3 Tate curve

The Tate curve and semistability.

4.6 Weak admissibility

4.6.1 Hodge and Newton polygons

Definitions, comparisons.

4.6.2 Weak admissibility

Weak admissibility, classification in low dimension.

4.6.3 D_{st} is weakly admissible

Why semistable representations give weakly admissible modules.

4.7 Fundamental sequence

4.7.1 The exact sequence

The fundamental exact sequences and log.

4.7.2 Extensions

Extensions of semistable and crystalline representations.

5 \mathbb{Q}_p -representations

5.1 Etale φ -modules

5.1.1 Torsion representations

Admissibility and classification.

5.1.2 Integral representations

Admissibility and classification.

5.1.3 Rational representations

Admissibility and classification.

5.2 Fields of norms

5.2.1 Overview of construction

Overview.

5.2.2 Galois equivalence

From characteristic 0 to characteristic p .

5.3 (φ, Γ) -modules

Attaching (φ, Γ) -modules to \mathbb{Q}_p -representations.

6 Differential Equations and Admissibility

6.1 Modules over the open disc

6.1.1 p -adic Analysis

Properties of the ring of converging power series over the open disc.

6.1.2 Filtered modules and modules over the open disc

From filtered modules with Frobenius and monodromy to modules over the open disc.

6.1.3 The Robba ring

The Robba ring, slopes.

6.1.4 Weak admissibility

Weak admissibility and slope 0.

6.2 \mathfrak{S} -modules

6.2.1 Slope 0

An equivalence of (isogeny) categories.

6.2.2 Fields of norms

Relation to fields of norms of a deeply ramified extension.

6.3 Weak admissibility and admissibility

Proof of the equivalence.