Math 1a Section 1

Homework 3

Due noon, Monday, October 24, 2011

All numbered exercises are from the textbook, Calculus by Apostol.

1. 10.4.10
2. Let $(z_n)_{n\geq 1}$ be a sequence of complex numbers. Show that $(z_n)_{n\geq 1}$ converges with $\lim_{n\to\infty} z_n = 0$ if and only if the sequence $(|z_n|)_{n\geq 1}$ converges with $\lim_{n\to\infty} |z_n| = 0$.
3. Use the previous exercise and what you learned in class to show that if z is a complex number with $|z| < 1$ then the sequence $(a_n)_{n\geq 1}$ where $a_n = 1 + z + z^2 + \cdots + z^n$ converges, and compute its limit. [Hint: can you recall what we proved in class about the expression a_n?]
4. Find the domain of definition of the function
$$f(x) = \sqrt{x^2 - 2} + \frac{1}{x^2 - 4}$$
taking the real variable x to the set of real numbers.
5. Show that the function $f : [2, \infty) \to [4, \infty)$ defined by $f(x) = x^2 - 4x + 8$ is bijective and compute its inverse $f^{-1} : [4, \infty) \to [2, \infty)$.