Math 1a Section 1

Homework 6

Due noon, Monday, November 21, 2011

All numbered exercises are from the textbook, Calculus by Apostol.

1. For $r > 0$ compute $\lim_{n\to\infty} n(r^{\frac{1}{n}} - 1)$.
2. Let $0 < a < b$ and T_n the following partition:

$$T_n = \left[a, a\left(\frac{b}{a}\right)^{\frac{1}{n}}\right] \cup \left[a\left(\frac{b}{a}\right)^{\frac{1}{n}}, a\left(\frac{b}{a}\right)^{\frac{2}{n}}\right] \cup \cdots \cup \left[a\left(\frac{b}{a}\right)^{\frac{j-1}{n}}, a\left(\frac{b}{a}\right)^{\frac{j}{n}}\right] \cup \ldots \cup \left[a\left(\frac{b}{a}\right)^{\frac{n-1}{n}}, b\right]$$

   (a) Show that $f : [a, b] \to \mathbb{R}$ given by $f(x) = \dfrac{1}{x}$ is integrable.

   (b) Compute $\lim_{n\to\infty} L(f, T_n)$ and deduce that

$$\int_a^b \frac{dx}{x} = \log b - \log a$$

3. 2.4.1
4. 2.4.15
5. 1.15.7