Homework 2

Due Wednesday, April 17, at noon

You are encouraged to work together with others, but you must write up the solutions on your own. All numbered exercises are from Dummit and Foote, third edition.

1. (5pt) 13.2.14
2. (5pt) (Restatement of 13.4.5) Let F be a field and let $P \in F[X]$ with splitting field E/F.
   (a) Show that for any element α of some extension of F, $E(\alpha)$ is a splitting field of P over $F(\alpha)$.
   (b) Show that every irreducible polynomial $Q \in F[X]$ with a root in E has all roots in E. [Hint: Read the hint from the book.]
3. (5pt) 13.4.6
4. (5pt) Let α and β be two algebraic elements over a field F. Assume that the degree of the minimal polynomial of α over F is relatively prime to the degree of the minimal polynomial of β over F. Prove that the minimal polynomial of β over F is irreducible over $F(\alpha)$.
5. (5pt) Let E and K be finite field extensions of F such that $[EK : F] = [E : F][K : F]$. Show that $K \cap E = F$.