Homework 7

Due Wednesday, May 29, at noon

You are encouraged to work together with others, but you must write up the solutions on your own. All numbered exercises are from Dummit and Foote, third edition.

1. 14.6.35
2. 14.6.43
3. 14.6.50 a-c.
4. Determine the Galois groups of the following polynomials in $\mathbb{Q}[X]$: $X^4 - 25, X^4 + 4, X^4 + 2X^2 + X + 3, X^5 + X - 1, X^5 + 20X + 16$. [Hint: A_5 is generated by a cycle of length 3 and a cycle of length 5.]
5. Let p be a prime number. A finite extension of fields K/F is said to be a p-extension if $[K : F]$ is a power of p.
   (a) Suppose K/F is a Galois p-extension and L/K is another Galois p-extension. Let E/L be any extension such that E/F is Galois. Show that there exists a Galois p-subextension E_p/L of E/L which is maximal among the Galois p-subextensions of E/L.
   (b) Keep the notation from part a. Show that E_p/F is Galois and deduce that the Galois closure of L/F is a p-extension of F.
   (c) Give an example of a (necessarily non-Galois) p-extension K/F and a Galois p-extension L/K such that the Galois closure of L/F is not a p-extension of F.