Ultraviolet catastrophe

The ultraviolet catastrophe, also called the Rayleigh–Jeans catastrophe, was the prediction of late 19th century/early 20th century classical physics that an ideal black body at thermal equilibrium will emit radiation in all frequency ranges, emitting more energy as the frequency increases. By calculating the total amount of radiated energy, (i.e., the sum of emissions in all frequency ranges), it can be shown that a blackbody would release an infinite amount of energy, contradicting the principles of conservation of energy and indicating that a new model for the behaviour of blackbodies was needed.

The term "ultraviolet catastrophe" was first used in 1911 by Paul Ehrenfest, but the concept originated with the 1900 derivation of the Rayleigh–Jeans law. The phrase refers to the fact that the Rayleigh-Jeans law accurately predicts experimental results at radiative frequencies below $10^5$ GHz, but begins to diverge with empirical observations as these frequencies reach the ultraviolet region of the electromagnetic spectrum.[1] Since the first appearance of the term, it has also been used for other predictions of a similar nature, as in quantum electrodynamics and such cases as ultraviolet divergence.

Figure 1. The ultraviolet catastrophe is the error at short wavelengths in the Rayleigh–Jeans law (depicted as "classical theory" in the graph) for the energy emitted by an ideal black-body. The error, much more pronounced for short wavelengths, is the difference between the black curve (as classically predicted by the Rayleigh–Jeans law) and the blue curve (the measured curve as predicted by Planck's law).
Problem

The ultraviolet catastrophe results from the equipartition theorem of classical statistical mechanics which states that all harmonic oscillator modes (degrees of freedom) of a system at equilibrium have an average energy of $\frac{1}{2} kT$, i.e., equipartition of energy.

An example, from Mason's *A History of the Sciences,*\footnote{2} illustrates multi-mode vibration via a piece of string. As a natural vibrator, the string will oscillate with specific modes (the standing waves of a string in harmonic resonance), dependent on the length of the string. In classical physics, a radiator of energy will act as a natural vibrator. And, since each mode will have the same energy, most of the energy in a natural vibrator will be in the smaller wavelengths and higher frequencies, where most of the modes are.

According to classical electromagnetism, the number of electromagnetic modes in a 3-dimensional cavity, per unit frequency, is proportional to the square of the frequency. This therefore implies that the radiated power per unit frequency should follow the Rayleigh–Jeans law, and be proportional to frequency squared. Thus, both the power at a given frequency and the total radiated power is unlimited as higher and higher frequencies are considered: this is clearly unphysical as the total radiated power of a cavity is not observed to be infinite, a point that was made independently by Einstein and by Lord Rayleigh and Sir James Jeans in 1905.

Solution

Planck derived the correct form for the intensity spectral distribution function by making some strange (for the time) assumptions. In particular, Planck assumed that electromagnetic radiation can only be emitted or absorbed in discrete packets, called quanta, of energy: $E_{\text{quanta}} = h \nu = h \frac{c}{\lambda}$, where $h$ is Planck's constant. Planck's assumptions led to the correct form of the spectral distribution functions: $B_\lambda(\lambda,T) = \frac{2 h c^2}{\lambda^5} \left( e^{\frac{h c}{(\lambda k_B T)}} - 1 \right)$. \footnote{3} Albert Einstein solved the problem by postulating that Planck's quanta were real physical particles—what we now call photons, not just a mathematical fiction. He modified statistical mechanics in the style of Boltzmann to an ensemble of photons. Einstein's photon had an energy proportional to its frequency and also explained an unpublished law of Stokes and the photoelectric effect.\footnote{3}

Historical inaccuracies

Many popular histories of physics, as well as a number of physics textbooks, present an incorrect version of the history of the ultraviolet catastrophe. In that version, the "catastrophe" was first noticed by Planck, who developed his formula in response. In fact Planck never concerned himself with this aspect of the problem, because he did not believe that the equipartition theorem was fundamental – his motivation for introducing "quanta" was entirely different. That Planck's proposal happened to provide a solution for it was realized only later, as stated above.\footnote{4} Though the true sequence of events has been known to historians for many decades, the historically
incorrect version persists, in part because Planck's actual motivations for the proposal of the quantum are complicated and difficult to summarize for a lay audience.[5]

References[edit]


5. For some of the historiographical debates over what actually motivated Planck, see
