ISSUES TO ADDRESS...

- **Stress and strain**: What are they and why are they used instead of load and deformation?

- **Elastic behavior**: When loads are small, how much deformation occurs? What materials deform least?

- **Plastic behavior**: At what point do dislocations cause permanent deformation? What materials are most resistant to permanent deformation?

- **Toughness and ductility**: What are they and how do we measure them?
Vocabulary: Mechanical Properties of Materials

- The changes in materials dimensions in response to mechanical forces is called **deformation**.

- If upon removal of load the material reverts back to its initial size – **elastic** deformation.

- If application and removal of the load results in a permanent material’s shape change – **plastic** deformation.

- **Fracture** occurs when a structural component separates into two or more pieces.

- Material **failure**, i.e. an inability of a component to perform its desired function, may occur prior to fracture.

- Materials behavior (e.g. failure) depends on load **nature** its **time** schedule, and **environmental conditions** (e.g. temperature).
Tensile Stress-Strain Test

Conditions are defined by ASTM Standards:
- load-time schedule
- temperature
- sample shape
Strain-Stress Concept

To compare specimens of different sizes, the \textbf{load} is calculated per unit area.

**Engineering tensile stress:** \( \sigma = \frac{F}{A_0} \)

F is load applied perpendicular to the specimen cross section; \( A_0 \) is the cross-sectional area \textbf{before} application of the load.

• **Engineering tensile strain:** \( \varepsilon = \frac{\Delta l}{l_0} \times 100 \% \)

These definitions of stress and strain allow to compare test results for specimens of \textbf{different} cross-sectional area \( A_0 \) and of \textbf{different initial} length \( l_0 \).

Stress and strain are \textbf{positive} for tensile loads and \textbf{negative} for compressive loads.
Types of Deformation (1)

Types of loading are defined by the direction of applied forces.

- **Elongation** – *positive* linear strain
- **Contraction** – *negative* linear strain

Forces applied normal to the sample surface.

**Tensile** load

**Compressive** load
Types of Deformation (2)

- Shear strain is \( q \) to the strain angle \( q \)
- Torsional deformation is \( f \) to the twist angle, \( f \)

**Pure shear** load:
- Applied force is parallel to both sample faces

**Torsion** is a variation of pure shear
- Torsion load produces a rotational motion (twist) around the axis of symmetry
Torsion Test

Angular Displacement Gage to measure the relative angle of rotation

The torsion machine is equipped with a linear variable-differential transformer to measure the twisting moment applied to the specimen.
ELASTIC DEFORMATION

1. Initial

2. Small load
   - Bonds stretch

3. Unload
   - Return to initial

Elastic means reversible!

- Linear-elastic
- Non-Linear-elastic

Strain

F

δ
1. Initial
2. Small load: bonds stretch & planes shear
3. Unload: planes still sheared

Plastic means permanent!
**ENGINEERING STRESS**

- **Tensile stress, \( \sigma \):**
  \[
  \sigma = \frac{F_t}{A_o}
  \]
  Load applied *perpendicular* to the specimen cross section

- **Shear stress, \( \tau \):**
  \[
  \tau = \frac{F_s}{A_o}
  \]
  \( F_s \) is a load constituent *parallel* to the specimen cross section

Stress has units: N/m\(^2\) or lb/in\(^2\)

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- **ENGINEERING STRESS**

- **Shear stress, \( \tau \):**
  \[
  \tau = \frac{F_s}{A_o}
  \]
  \( F_s \) is a load constituent *parallel* to the specimen cross section

Stress has units: N/m\(^2\) or lb/in\(^2\)
**ENGINEERING STRAIN**

- **Tensile strain:**
  \[
  \varepsilon_z = \frac{L - L_0}{L_0} \times 100\%
  \]

- **Lateral strain:**
  \[
  \varepsilon_{x,y} = \frac{d - d_0}{d_0} \times 100\%
  \]

- **Shear strain:**
  \[
  \gamma = \frac{\Delta L}{L_0} \times 100\% \quad \text{or} \quad \tan \gamma \times 100\%
  \]

Strain is always dimensionless.
Stress Versus Strain: Elastic Deformation

Typical Stress-Strain Diagram for one-dimensional tensile test

Hooke's Law:  \( \sigma = E \varepsilon \)

\( E [N/m^2; GPa] \) is Young's modulus or modulus of elasticity
Reminder:
Atomic Mechanism of Elastic Deformation

\[ E = \frac{1}{r_0} \cdot (\frac{dF}{dr})_{r_0} \]

Weaker bonds – the atoms easily move out from equilibrium position
Reminder: PROPERTIES FROM BONDING

- Elastic modulus, $E$

The “stiffness” of the bond is given by:

$$S = \frac{dF}{dr} = d\frac{d}{dr} U$$

$$S_0 \approx \left(\frac{d^2 U}{dr^2}\right)_{r_0}$$

$$E = \frac{S_0}{r_0}$$
Modulus of Elasticity for Different Metals
Young’s modulus is a numerical constant, named for the 18th-century English physician and physicist Thomas Young, that describes the elastic properties of a solid undergoing tension or compression in only one direction.

Higher $E$ – higher “stiffness”
**Stress Versus Strain: Constitutive Relations**

*Cauchy* generalized Hooke's law to three dimensional elastic bodies and stated that the 6 components of stress are linearly related to the 6 components of strain. The stress-strain relationship written in matrix form, where the 6 components of *stress* and *strain* are organized into column vectors:

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{zx} \\
\varepsilon_{xy}
\end{bmatrix}
= \begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\
S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\
S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\
S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{zx} \\
\varepsilon_{xy}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{yz} \\
\sigma_{zx} \\
\sigma_{xy}
\end{bmatrix}
= \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{yz} \\
\varepsilon_{zx} \\
\varepsilon_{xy}
\end{bmatrix}
\]

\[\varepsilon = S \cdot \sigma\]

\[\sigma = C \cdot \varepsilon\]

where \(C\) is the **stiffness matrix**, \(S\) is the **compliance matrix** \((S = C^{-1})\)

There are 36 stiffness matrix components. However, the conservative materials possess a strain energy density function and as a result, the stiffness and compliance matrices are symmetric and thus only 21 stiffness components are actually independent in Hooke's law.
Elastic Deformation: Non-linear Behavior

For some materials (typically for polymers, also concrete) deformation is *not linear* even on the initial stage of S-S diagram, BUT still *reversible*, i.e. elastic!!
Elastic Deformation: Poisson’s Ratio

For anisotropic material $\nu=0.25$
Typical value:
Metals $\sim0.28-0.34$
Ceramics $\sim0.25$
Polymers $\sim0.40$

Materials subject to tension (compression) also **shrink (bulge) laterally.**

\[ \varepsilon_z = \frac{\Delta L_z}{L_{oz}} \]
\[ \varepsilon_{x,y} = \frac{\Delta L_{x,y}}{L_{ox,y}} \]

**Poisson’s Ratio:**
\[ \nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\varepsilon_y}{\varepsilon_z} \]

By definition $\nu$ is a positive dimensionless material characteristic, while $\varepsilon_{x,y}$ and $\varepsilon_z$ are always appositive.
Elastic Deformation: Shear Modulus

Relationship of shear stress to shear strain:
\[ \tau = G \gamma, \]
where: \( \gamma = \tan \theta = \frac{\Delta y}{z_0} \) and \( G \) is Shear Modulus (Units: N/m\(^2\))

For isotropic material:
\[ E = 2G(1+\nu) > G \sim 0.4E \]

(Note: most materials are elastically anisotropic: the elastic behavior varies with crystallographic direction, see Chapter 3)
**Elastic Deformation: Bulk Modulus**

- **Elastic Bulk modulus, \( K \):**
  
  \[
  P = -K \frac{\Delta V}{V_0}
  \]

- Special relations for isotropic materials:
  
  \[
  K = \frac{E}{3(1-2v)}
  \]

- **Pressure test:**
  
  \( V_0 \)- initial volume
  
  \( \Delta V \)- vol. change
Stress Versus Strain: Plastic Deformation

Irreversible deformation begins

Region of Plastic Deformation
For most metals elastic deformation persists to strain only of ~0.5%.

Beyond some critical point permanent non-recoverable deformation, i.e. yielding, occurs.

On the atomic levels yielding means *breaking bonds* with original atoms.

Yielding mechanisms are different for crystalline (Chapter 7) and amorphous (Chapter 12) materials.
**YIELD STRENGTH, \( \sigma_y \) (1)**

- \( \sigma_y \) is a stress at which *noticeable* (\( \varepsilon_p = 0.2 \% \); by convention) plastic (permanent) strain has occurred.

- The yield stress is a measure of resistance to plastic deformation.

- *Proportional limit* (yield point \( P \)) – S-S curve starts to deviate from linearity.
• For some steels (low carbon) the S-S curve posses a *yield point phenomenon*, which is characterized by an upper and lower yield points.

• In this case $\sigma_y$ is taken at the *lower yield* point.
YIELD STRENGTH: COMPARISON

- **σ_y(ceramics)**
  - Steel (4140)\text{qt}
  - Ti (5Al-2.5Sn)\text{ag}
  - W (pure)
  - Cu (71500)\text{cw}
  - Mo (pure)
  - Steel (4140)\text{a}
  - Steel (1020)\text{cd}
- **σ_y(metals)**
  - Ti (pure)\text{a}
  - Ta (pure)
  - Cu (71500)\text{hr}
  - Al (6061)\text{ag}
- **σ_y(polymers)**

**Room T values**

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

Yield strength, $\sigma_y$ (MPa)

- 2000
- 1000
- 500
- 200
- 100
- 70
- 60
- 50
- 40
- 30
- 20
- 10
- 1

Metals/Alloys

Graphite/Ceramics/Semiconductors

Polymers

Composites/fibers

Hard to measure, since in tension, fracture usually occurs before yield.

Hard to measure, since in ceramic matrix and epoxy matrix composites, since in tension, fracture usually occurs before yield.
Tensile Strength, TS

- Maximum possible engineering stress in tension (point M).

- Metals: occurs when noticeable necking starts.
- Ceramics: occurs when crack propagation starts.
- Polymers: occurs when polymer backbones are aligned and about to break.
TENSILE STRENGTH: COMPARISON

- **Metals/Alloys**
  - Steel (4140)\textsuperscript{qt}
  - W (pure)
  - Ti (5Al-2.5Sn)\textsuperscript{a}
  - Steel (4140)\textsuperscript{a}
  - Cu (71500)\textsuperscript{hr}
  - Cu (71500)\textsuperscript{cw}
  - Steel (1020)
  - Al (6061)\textsuperscript{ag}
  - Ti (pure)\textsuperscript{a}
  - Ta (pure)
  - Al (6061)\textsuperscript{a}

- **Graphite/Ceramics/Semicond**
  - Diamond
  - Si nitride
  - Al oxide
  - Si crystal
  - Glass-soda
  - Concrete
  - Graphite

- **Polymers**
  - Nylon 6,6
  - PC
  - PVC
  - PET
  - HDPE
  - LDPE

- **Composites/fibers**
  - CFRE (fiber)
  - GFRE (fiber)
  - AFRE (fiber)
  - UFRE (fiber)
  - Wood (fiber)

**Room T values**
- TS(\text{ceram})
- ~TS(\text{met})
- ~TS(\text{comp})
- >> TS(\text{poly})

- a = annealed
- hr = hot rolled
- ag = aged
- cd = cold drawn
- cw = cold worked
- qt = quenched & tempered

AFRE, GFRE, & CFRE = aramide, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.
Plastic tensile strain at failure:

Another ductility measure:

\%AR = \frac{A_o - A_f}{A_o} \times 100

\%EL = \frac{L_f - L_o}{L_o} \times 100

Note: %AR and %EL are often comparable, because crystal slip does not change material volume. However, if internal voids form in neck. %AR > %EL possible.
Temperature Effect on the S-S Diagram

$\sigma_y$, TS and E decrease with increasing temperature, while ductility increases
Mechanical Properties of Metals

<table>
<thead>
<tr>
<th>Metal Alloy</th>
<th>Yield Strength MPa (ksi)</th>
<th>Tensile Strength MPa (ksi)</th>
<th>Ductility, %EL [in 50 mm (2 in.)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>35 (5)</td>
<td>90 (13)</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>69 (10)</td>
<td>200 (29)</td>
<td>45</td>
</tr>
<tr>
<td>Brass (70Cu–30Zn)</td>
<td>75 (11)</td>
<td>300 (44)</td>
<td>68</td>
</tr>
<tr>
<td>Iron</td>
<td>130 (19)</td>
<td>262 (38)</td>
<td>45</td>
</tr>
<tr>
<td>Nickel</td>
<td>138 (20)</td>
<td>480 (70)</td>
<td>40</td>
</tr>
<tr>
<td>Steel (1020)</td>
<td>180 (26)</td>
<td>380 (55)</td>
<td>25</td>
</tr>
<tr>
<td>Titanium</td>
<td>450 (65)</td>
<td>520 (75)</td>
<td>25</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>565 (82)</td>
<td>655 (95)</td>
<td>35</td>
</tr>
</tbody>
</table>

Note: The Yield and Tensile Strengths of material depend on its prior mechanical and thermal treatments, impurities level, etc., while elastic modulus is relatively insensitive to these effects.
TOUGHNESS (Chapter 8)

- **Energy to break a unit volume of material**
- Approximate by the area under the stress-strain curve.
- Units: \([J/m^3]\)
True Stress and Strain

- **True stress**, $\sigma_T$, is defined by using instantaneous cross-sectional area of the sample, $A_i$

\[ \sigma_T = \frac{F}{A_i} \]

True Strain:

\[ \varepsilon_T = \ln\left(\frac{l_i}{l_o}\right) \]
True Stress and Strain

\[ \sigma_T = \frac{F}{A_i} \]

\[ \varepsilon_T = \ln \left( \frac{l_i}{l_o} \right) \]

If volume of materials remains constant (until necking point):

\[ A_i l_i = A_o l_o \]

Then the following relations hold:

\[ \sigma_T = \sigma (1 + \varepsilon) \] and \[ \varepsilon_T = \ln (1 + \varepsilon) \]

The “corrected” true S-S curve accounts for the complex stress state within the neck region.
• Resistance to permanently indenting the surface.
• Large hardness means:
  -- resistance to plastic deformation or cracking in compression.
  -- better wear properties.

HARDNESS

e.g., 10mm sphere

apply known force (1 to 1000g)

measure size of indent after removing load

Smaller indents mean larger hardness.

most plastics
brasses
Al alloys
easy to machine
steels
file hard
cutting
tools
nitrided
steels
diamond

increasing hardness
### Hardness-Testing Methods

- A qualitative Mohs scale: ability of a material to scratch another material: from 1 (softest, e.g. talc) to 10 (hardest, i.e. diamond)

- A variety of quantitative hardness test methods:

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number[^a]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td><img src="image1" alt="Brinell Diagram" /></td>
<td>$P$</td>
<td>$\text{HB} = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$</td>
</tr>
<tr>
<td>Vickers microhardness</td>
<td>Diamond pyramid</td>
<td><img src="image2" alt="Vickers Diagram" /></td>
<td>$P$</td>
<td>$\text{HV} = 1.854P/d_1^2$</td>
</tr>
<tr>
<td>Knoop microhardness</td>
<td>Diamond pyramid</td>
<td><img src="image3" alt="Knoop Diagram" /></td>
<td>$P$</td>
<td>$\text{HK} = 14.2P/l^2$</td>
</tr>
<tr>
<td>Rockwell and Superficial Rockwell</td>
<td><img src="image4" alt="Rockwell Diagram" /></td>
<td>60 kg, 100 kg, 150 kg, 15 kg, 30 kg, 45 kg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hardness Tests

Rockwell hardness tester

Hardness Scales
• Design uncertainties mean we do not push the limit.
• **Factor of safety, N**

\[ \sigma_{\text{working}} = \frac{\sigma_y}{N} \]

Often N is between 1.2 and 4

• **Ex:** Calculate a diameter, d, to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

\[
\frac{220,000 \text{N}}{\pi \left( \frac{d^2}{4} \right)} = \frac{\sigma_y}{5}
\]

1045 plain carbon steel:
\( \sigma_y = 310 \text{MPa} \)
\( \text{TS} = 565 \text{MPa} \)

\( F = 220,000 \text{N} \)
• **Stress and strain**: These are size-independent measures of load and displacement, respectively.

• **Elastic behavior**: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus (E or G).

• **Plastic behavior**: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches $\sigma_y$.

• **Toughness**: The energy needed to break a unit volume of material.

• **Ductility**: The plastic strain at failure.