Math 444/539 : Geometric Topology
Problem Set 11

Everyone should do all the problems.

Problems :

1. Construct nonnormal covering spaces of the Klein bottle by a Klein bottle and by a torus.

2. For a path-connected, locally path-connected, and semilocally simply-connected space X, call a path-connected covering space $\rho : \tilde{X} \to X$ *abelian* if it is normal and has abelian deck transformation group. Show that X has an abelian covering space that is a covering space of every other abelian covering space of X, and that such a "universal" abelian covering space is unique up to isomorphism. Describe this covering space explicitly for X the wedge of 2 circles.

3. Draw the Cayley graphs of the following groups.

   (a) The dihedral group of order 8. You can choose any generating set you like.

   (b) The group $\mathbb{Z} * \mathbb{Z}/2 \cong \langle a, b \mid b^2 \rangle$. Use the generating set $\{a, b\}$.

   (c) The symmetric group S_3. Use the generating set $\{(1, 2), (2, 3)\}$.

4. Show that the normal covering spaces of the wedge of two circles are precisely the graphs that are Cayley graphs of groups with two generators.

5. Show that a finitely generated group has only a finite number of subgroups of a given finite index. Hint : first do the case of free groups, using covering spaces of graphs. Then prove the general using the fact that every group is the quotient group of a free group.