Math 444/539 : Geometric Topology
Problem Set 12

Everyone should do all the problems.

Problems :

1. Let G be a group and let X be the $K(G,1)$ constructed in class (by gluing together simplices – the construction is also described as Example 1B.7 in Hatcher's book, which is available on his webpage). Complete the proof in class that X is a $K(G,1)$ by proving that the action of G on X is a covering space action.

2. Let X be a connected CW complex and G be a group such that every homomorphism $\pi_1(X) \to G$ is trivial. Let Y be a $K(G,1)$. Prove that every map $X \to Y$ is null-homotopic.

3. Let G be the fundamental group of a graph of groups such that every vertex group is trivial. Prove that G is a free group.

4. Let G be the fundamental group of the following graph of groups. The graph has vertices $v_1, v_2, v_3, \ldots$ and directed edges $e_1, e_2, e_3, \ldots$ with e_i connecting v_i with v_{i+1}. For each i, let $G(v_i) = \mathbb{Z}$. Finally, the homomorphism $G(v_i) \to G(v_{i+1})$ associated to the directed edge e_i is multiplication by i. Prove that $G \cong \mathbb{Q}$.

5. The following was one of the initial motivations for the theory of graphs of groups. Do the following without using the theory of graphs of groups.

   (a) Let F_n be a free group on n letters. Prove that F_n acts freely on a tree.

   (b) Let G be a group that acts freely on a tree. Prove that G is a free group.

   (c) Use part b to give an alternate proof of the fact that a subgroup of a free group is free.