Math 366 : Geometry
Problem Set 9

1. Let $X \subset \mathbb{R}^2$ be a finite set of points. Prove that each Voronoi cell of X is convex.

2. For each $n \geq 4$, construct a set X of n points in $\mathbb{R}^2$ such that one of the Voronoi cells of X is an $(n-1)$-gon. Prove that your answer works!

3. Let $X \subset \mathbb{R}^2$ be a finite set of $n \geq 3$ points that do not all line on a single line. Prove that there exists a triangulation with vertex set X. Hint : Prove it by induction on n. The base case $n = 3$ is easy. For the inductive case, choose a vertex from X, remove it, and see what happens. Be careful – you might get a set of points that all lie on a line!

4. Let $X \subset \mathbb{R}^2$ be the vertices of a convex n-gon (remark : X is not in general position!). Prove that any two triangulations of X are connected by at most $2n$ flips. Hint : Try to connect a given triangulation to one where all the triangles contain some fixed vertex.

5. Let $X \subset \mathbb{R}^2$ be a finite set in general position and let A be a triangulation of X. Assume that none of the triangles in X is obtuse. Prove that A is the Delaunay triangulation.