Math 444/539, Homework 8

1. Construct a 2-dimensional CW complex that contains both an annulus $S^1 \times I$ and a Möbius band as deformation retracts.
2. Prove that S^∞ is contractible.
3. Given positive integers v and e and f satisfying $v - e + f = 2$, construct a CW complex structure on S^2 having v 0-cells, e 1-cells, and f 2-cells.
4. Let X be the space quotient space of S^2 obtained by identifying the north and south poles to a single point.
   (a) Construct an explicit CW complex structure on X.
   (b) Use this CW complex structure to calculate the fundamental group of X.
5. Consider the quotient space of a cube $[0, 1]^3$ obtained by identifying each square face with the opposite square face via the right-handed screw motion consisting of a translation by one unit in the direction perpendicular to the face combined with a one-quarter twist of the face about its center point. Show this quotient space X is a CW complex with two 0-cells, four 1-cells, three 2-cells, and one 3-cell. Using this structure, show that $\pi_1(X)$ is isomorphic to the quaternion group $\{\pm 1, \pm i, \pm j, \pm k\}$ of order eight.