

# Math 30810: Honors Algebra III

## Problem Set 2

Do 10 of the following 14 problems. Any extra problems you solve are extra credit.

- Chapter 2, problems 4.2, 4.3, 4.4, 4.7, 4.10, 5.1, 5.2, 5.5, 5.6, 6.3, 6.8, 6.9, 6.11.

Additional problem (with two parts):

- (a) Prove that  $S_n$  is generated by transpositions by induction on  $n$ . Here is a hint:
- The group  $S_{n-1}$  is a subgroup of  $S_n$  in a natural way. Argue that it is enough to show that for any  $\sigma \in S_n$ , there exists a transposition  $\tau \in S_n$  such that  $\tau\sigma \in S_{n-1}$ .
- (b) Prove that  $S_n$  is generated by *adjacent* transposition (i.e. those of the form  $(i, i+1)$  for some  $1 \leq i < n$ ) by showing how to express an arbitrary transposition as a product of adjacent ones.