1 ACCELERATION OF A FREELY-FALLING BODY

Reference: Halliday, Resnick & Walker chapters 2 & 4

Physical principle to be studied: Your text tells you that the acceleration of a body falling freely near the surface of the earth is constant and downward (toward the center of the earth) with a magnitude of 9.8 m/s\(^2\). In this experiment you will make your own measurement of the acceleration \(g\) of a freely-falling body, and check to see if your measured value is consistent with the accepted value of 9.8 m/s\(^2\).

Additional concepts: You will be introduced to error estimation, calculation, and reporting. All of these are essential to the success of your lab as they will determine your ability to assess whether your measured value of \(g\) is consistent with the accepted value.

Before entering the lab: Refamiliarize yourself with the relationships between acceleration and velocity and between average velocity and instantaneous velocity. Remember what relationship the slope of a line represents between the axis variables. Review Measurements and Errors, paying special attention to “Interpolation,” “Significant figures and rounding off” points 1 and 2, “Estimated errors,” and “Lesson 3: Graphical Analysis to Verify Algebraic Equations.” Complete “Measurement and Error Helper Quiz 1” and be ready to approach your TA with any questions.

Equipment: (See figure 1.) Vertical stand, electromagnet, spark timer, weight, paper tape, tape measure and/or meter stick.
Procedure:
1. The stand must be level so that the wires are perfectly vertical. This ensures that as the weight falls, the metal ring on it always remains close to the wires without touching them. It is also important that the weight not be swinging back and forth when the magnet is turned off and the weight released. As a check, you should release the weight once or twice with the spark timer turned off, to be sure that the weight falls properly into the cup at the bottom of the stand.

2. To make your measurements, one partner should operate the spark timer while the other controls the electromagnet. Be sure that a fresh strip of paper is in place and that the magnet wires are well away from the spark wires. Beginning at the bottom, smooth the paper tape and fasten it snugly against the column of the stand with small pieces of masking tape.

3. Suspend the weight from the magnet, turn on the spark voltage, drop the weight, and turn off the spark voltage as soon as the weight has fallen into the cup.

4. Remove the paper strip and examine it carefully to see if a regular succession of points can be seen. A point may be missing here or there, but there should be no long gaps. You should be able to tell from the size of a gap whether or not any points are missing and, if so, how many are missing. If you are unable to determine with confidence how many points are missing, you will have to repeat the measurement with a fresh piece of paper.

Data collection and recording:
5. Tape your paper strip to your laboratory table and place a tape measure or meter stick on it so that you can easily measure the positions of the spark points.

6. Number the points consecutively beginning at the top where the weight was moving most slowly and the points are closest together. Do not number the very top point, which shows the location of the weight when the spark is turned on but before the magnet is turned off. You may even choose to ignore several of your first points, taking $t=0$ where your data first becomes clearly consistent. You can take $t=0$ at any time you choose, since you are interested only in the change in velocity with time. However, for whichever point you “start the clock,” you must then number the points consistently, making sure missing points are assigned their number so that you can calculate correctly the accumulated time corresponding to each point.

7. In order to determine the velocity of the weight during a given time interval between sparks, you will need to measure its displacement ($\Delta x$) during that interval. Measure the displacements between pairs of points and record (table 1 is provided at the end of this write up) values of $n$ and $\Delta x$ for at least 6 independent pairs of points (1&2, 3&4,…. not 1&2, 2&3,…). If you have fewer than 6 pairs, you will need to repeat the drop part of the experiment, if you have more pairs, choose a wide and roughly even distribution of pairs (1&2, 4&5, 9&10,… not 1&2, 3&4, 4&5,…).
8. For each measurement you make, you should make and record a corresponding estimate of the error. Laboratory error is primarily a function of the precision of your equipment and your ability to access that precision. In this experiment, your error arises from an uncertainty in $\Delta t$, the spark timing interval (given by the manufacturer as ±3%, so $0.03 \times (1/60)s = 5 \times 10^{-4} s$) and your ability to measure $\Delta x$, the distance between the marks on the paper strip. You must estimate how accurately you are able to measure the values of $\Delta x$, based on the precision of your ruler/meter stick and your ability to read it. If your meter stick is marked in mm, you should be able to read to mm and interpolate to a fraction of a mm (see Measurement and Error, “Interpolation”). However, if the point you are trying to measure is 2-3 mm in diameter, and odd shaped so that you cannot estimate the center, your error estimation may be greater. In this, and many other experiments, you may find that since you are trying to measure many similar things with the same instruments, you are tempted to use the same error estimation. In many cases, this is perfectly acceptable, but always remember to adjust your estimation when necessary.

**Before you leave the lab:** Make sure that table 1 is completed. This means that you will have to have completed some of the analysis described below. Make sure you understand how to complete the expected plot, how to obtain from it a value for $g$ with error and how to determine whether that value is consistent with the “book” value. Each lab partner must have the completed data sheet initialed by a Graduate Assistant.

**Analysis:**
9. Calculate the time corresponding to each velocity by recalling that for uniformly accelerated motion ($a =$ constant) the average velocity for a given time interval equals the instantaneous velocity at the center of the time interval.

10. As noted earlier, we have two error estimates, $\delta \Delta x$ and $\delta \Delta t$. The rule (see Measurement and Error, “Operations on Two or More Measured Quantities”) for combining relative errors when two quantities are multiplied or divided is

\[
\left(\frac{\delta v}{v}\right)^2 = \left(\frac{\delta (\Delta x)}{\Delta x}\right)^2 + \left(\frac{\delta (\Delta t)}{\Delta t}\right)^2
\]

Once you have these relative errors for the velocities, the absolute errors are obtained by multiplying them by the magnitude of the velocities. (In other words, solve for $\delta v$.)

11. Plot a graph of your average velocities, including error bars, versus time. If the acceleration of the falling weight was indeed a constant, the graph should show that the points are consistent with lying on a straight line and the slope of the line will be the magnitude of the acceleration, $a = \Delta v/\Delta t$.

12. Draw the best straight line you can through the plotted points. Also draw the lines having the largest and the smallest acceptable slopes. (Refer to Measurement and Error.) From the slopes of these three lines, determine the acceleration $g$ and the uncertainty in your value. Compare your result to the commonly used value for the magnitude of the acceleration due to gravity, $g = 9.80 \text{ m/s}^2$. Note that this “book value” of $g$ is reported.
without error because the error associated with it is much smaller than .01 m/s², which is the smallest for this precision. However, it doesn’t make sense for you to use g with higher precision because your own value cannot be expected to be more precise.

**Discussion:**

1) Are your measured value of g and the nominal value consistent? How do you decide whether or not they are consistent?

2) If your measured value was not consistent, give suggestions why you think it wasn’t. You cannot use “error in measuring” or “human error” as reasons because they’ve already been accounted for in ∆v. Are there any physical properties that your equations didn’t account for? Did anything “funny” happen during your execution of the lab?

3) What would you do next time to reduce the amount of error in your experiment, or do you think this is as good as it can get with this equipment?

4) Suppose the weight were somehow given an initial downward push instead of just being dropped. What effect, if any, would this have on the value of g you would obtain in the experiment? Why?

5) How can this experiment be improved? Write down 3 suggestions.
### Expt 1 - Table 1

<table>
<thead>
<tr>
<th>n pair</th>
<th>T (s) interval</th>
<th>t (s) average</th>
<th>δt (s)</th>
<th>Δx (m)</th>
<th>δΔx (m)</th>
<th>( v = \frac{\Delta x}{\Delta t} ) (m/s)</th>
<th>δv (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
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<td>5/120</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td>3/60</td>
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<td></td>
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