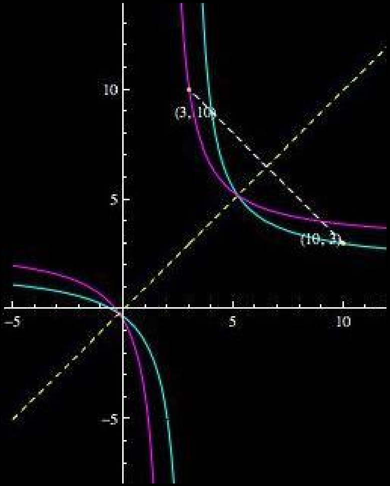
Graph of f^{-1}.

Since the equation $y = f^{-1}(x)$ is the same as the equation $x = f(y)$, the graphs of both equations are identical.

- To graph the equation $x = f(y)$, we note that this equation results from switching the roles of x and y in the equation $y = f(x)$.
- This transformation of the equation results in a transformation of the graph amounting to reflection in the line $y = x$.
- Thus the graph of $y = f^{-1}(x)$ is a reflection of the graph of $y = f(x)$ in the line $y = x$ and vice versa.
- **Note** The reflection of the point (x_1, y_1) in the line $y = x$ is (y_1, x_1). Therefore if the point (x_1, y_1) is on the graph of $y = f^{-1}(x)$, we must have (y_1, x_1) on the graph of $y = f(x)$.
- Not that this is the same as saying that $y_1 = f^{-1}(x_1)$ if and only if $x_1 = f(y_1)$.

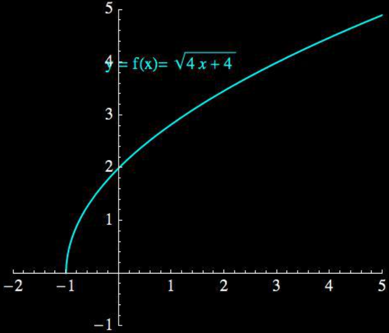
Graph of f^{-1}.

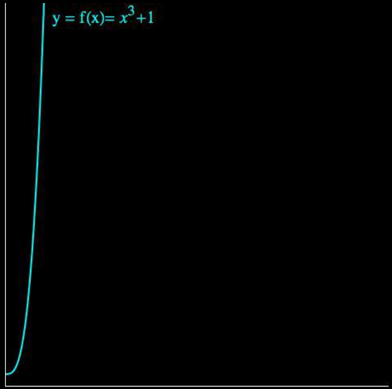
The graphs of $f(x) = \frac{2x+1}{x-3}$ (shown in blue) and $f^{-1}(x) = \frac{3x+1}{x-2}$ (shown in purple) are shown below.



Graph of f^{-1}.

Sketch the graphs of the inverse functions for $y = \sqrt{4x+4}$ and $y = x^3 + 1$ using the graphs of the functions themselves shown on the left and right below respectively.

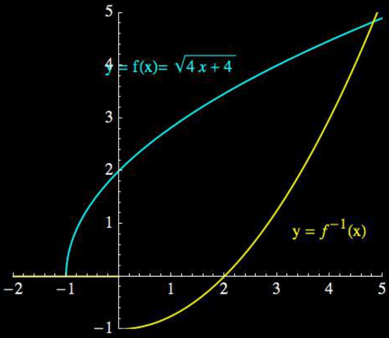


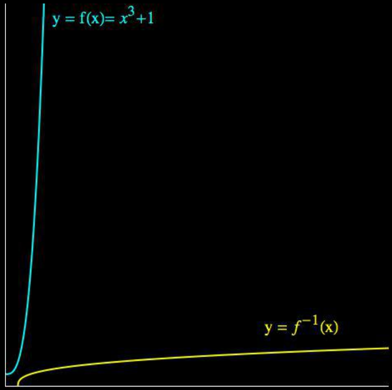


To sketch a graph of the inverse function you must draw the mirror image of the graph of the function itself in the line $y = x$.

Graph of f^{-1}.

We show the the graphs of the inverse functions for $y = \sqrt{4x+4}$ and $y = x^3 + 1$ in yellow below.



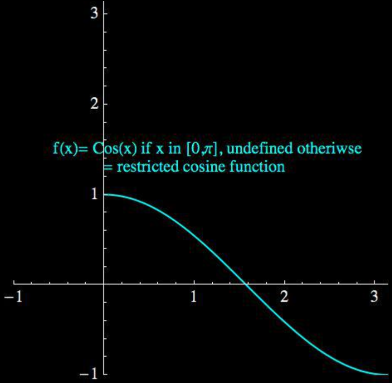


Restricted Cosine

Recall the restricted cosine function which was a one-to-one function defined as

$$f(x) = \begin{cases} \cos x & 0 \leq x \leq \pi \\ \\ \text{undefined} & \text{otherwise} \end{cases}$$

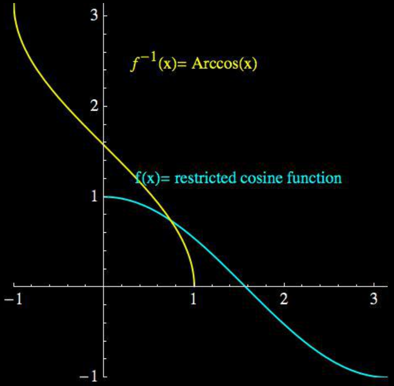
The graph of f is shown below.



Sketch the graph of $f^{-1}(x)$ known as $\arccos(x)$ or $\cos^{-1}(x)$.

Arccos(x) or Inverse Cosine

We show the the graphs of the inverse function for the restricted cosine function in yellow below. This function is referred to as arccos(x) or $\cos^{-1}(x)$.



- Note that the domain of arccos(x) is $[-1, 1]$ and its range is $[0, \pi]$.