Hyperlinks are shown in blue, download the `cdf player` *from the Wolfram Alpha website to view the* `Wolfram Alpha interactive demonstrations`*. When you have downloaded the cdf player, click on this symbol to view the demonstration.*

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$A \cap B = \{2, 4\}$ so $A \cap B \cap C = \{4\}$.

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1 is not in C and 8 is not in B so $A \cap B \cap D \cap C \subset D = \emptyset$.

Universal set and complements

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Venn Diagrams

We can also draw representations of two and three subsets of a universal set using Venn diagrams as shown below. The shaded regions below represent the given subsets of the universal set. (Note that in some cases, there are two set theoretic descriptions of the same set.)

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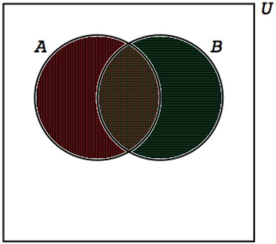
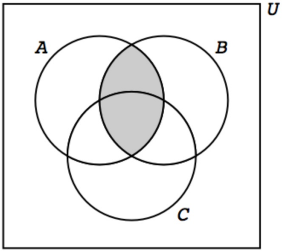
To find the shaded region corresponding to two sets A and B, you should shaded the sets A and B in different colors and the set $A \cap B$ will be the region where both shadings(colors) occur.



$A \cap B$



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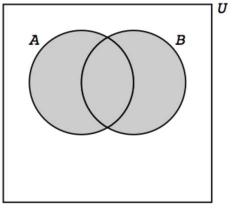

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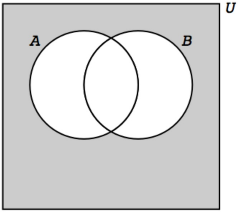

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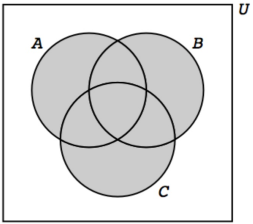
Venn Diagrams



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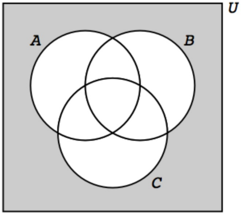

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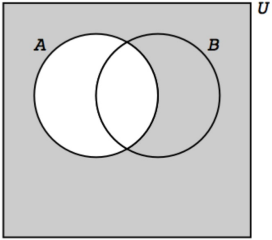
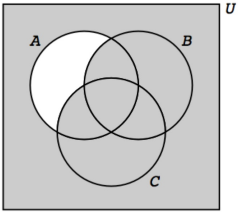


A'



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Venn Diagrams

The following interactive Venn diagram applet on Wolfram Alpha will allow you to experiment with identifying shaded regions of Venn diagrams:

Compute subsets

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Disjoint sets Two sets A and B are said to be disjoint if $A \cap B = \emptyset$. For example if $A = \{$All major league baseball players who got more than 700 home runs in their career$\} = \{$ Barry Bonds, Hank Aaron, Babe Ruth$\}$ and $B = \{$All major league baseball players with a career batting average greater than .350$\} = \{$ Ty Cobb, Rogers Hornsby, Joe Jackson$\}$, then A and B are disjoint, i.e. $A \cap B = \emptyset$.

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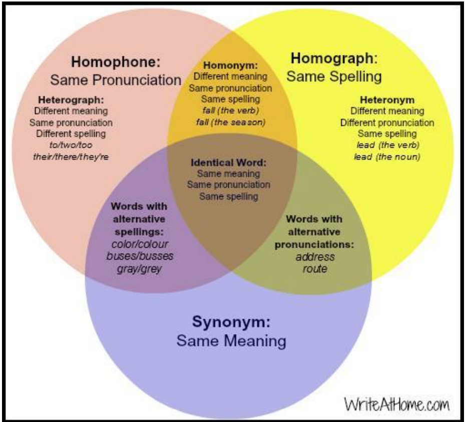
Note Complements have the following properties:

$$A \cap A' = \emptyset, \quad (A')' = A \quad A \cup A' = U$$

Venn diagrams for presentations

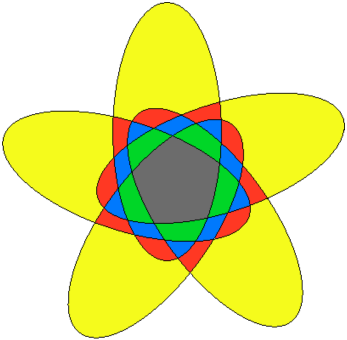
Venn diagrams of two or three sets are often used in presentations. Venn diagrams of more sets are possible, but tend to be confusing as a presentation tool because of the number of possible interactions.

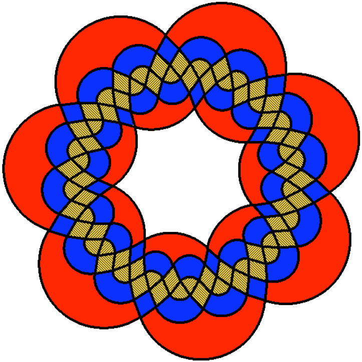


Venn Diagram of Word Relationships

Venn diagrams for presentations

With more than three sets it becomes difficult to extract much from the picture. The following diagrams show Venn diagrams for five sets on the left and for 7 sets on the right.





The Inclusion-Exclusion Principle

Definition For any finite set, S, we let $n(S)$ denote the number of objects in S.

The Inclusion Exclusion Principle If A and B are sets, Then

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}.$$

Example Check that the Principle is true for the following sets:

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$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$: $n(A \cup B) = 10$.

$A \cap B = \{5, 6, 7, \}$: $n(A \cap B) = 3$.
$n(A) = 7$, $n(B) = 6$.

$$10 = 7 + 6 - 3$$

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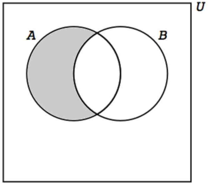
Formula 2 The shaded region below is $A \cap B^c$ and $(A \cap B^c) \cap (A \cap B) = \emptyset$ so

$$\boxed{n(A \cap B^c) = n(A) - n(A \cap B)}$$

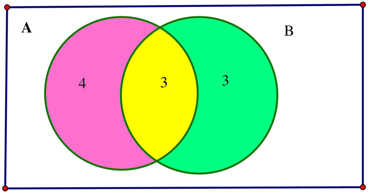


Venn diagrams and the Inclusion Exclusion Principle

We can sometimes use the inclusion-exclusion principle either as an algebraic or a geometric tool to solve a problem. We can use a Venn diagram to show the number of elements in each basic region to display how the numbers in each set are distributed among its parts.

Venn diagrams and the Inclusion Exclusion Principle

With $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{5, 6, 7, 8, 9, 10\}$ as above we saw that $n(A \cap B) = 3$, hence the 3 in the region of intersection, the yellow bit.



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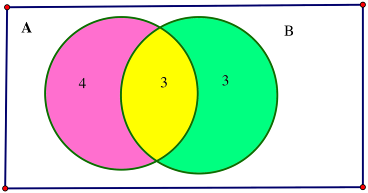

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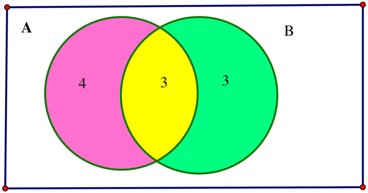

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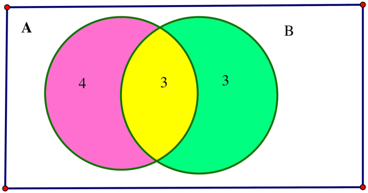
- Note $10 = n(A \cup B) = 4 + 3 + 3$.



More Inclusion-Exclusion

In general, the Inclusion-Exclusion Principle is an equation relating four numbers. Hence if you know three of them, you can work out the fourth.

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More Inclusion-Exclusion

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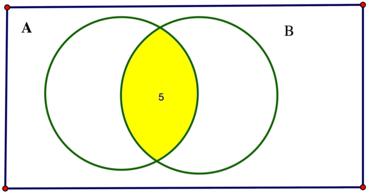

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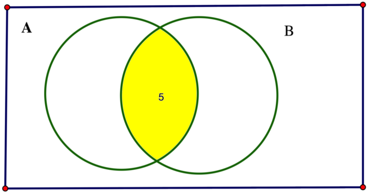
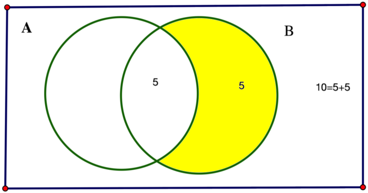
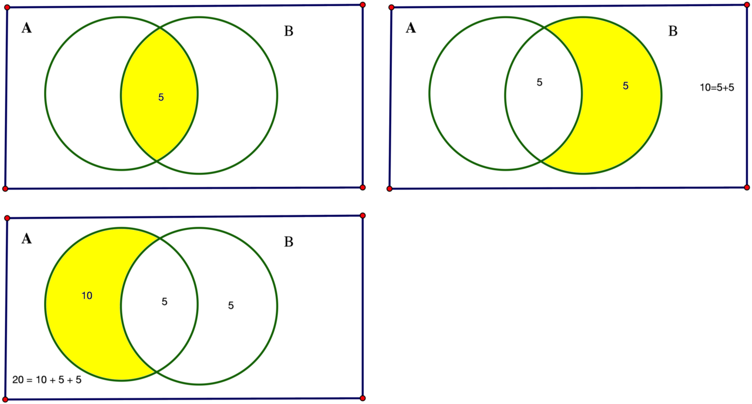




More Inclusion-Exclusion

Geometric:



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Example A survey of a group of students, revealed that 60 of them liked at least one of the cereals, Frosted Flakes or Lucky Charms. If 50 of them liked Frosted Flakes and 46 of them liked Lucky Charms,

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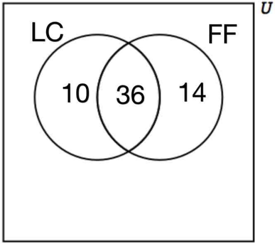

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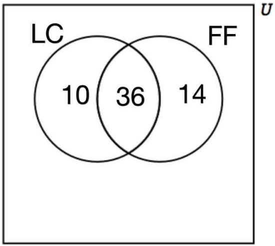

(c) How many students liked Frosted Flakes but did not like Lucky Charms?

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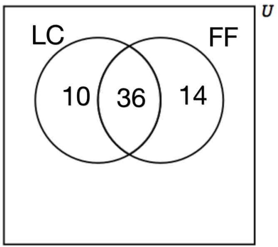

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$n(R \cup C \cup E) = 66$ and $n(U) = 68$, so by formula 1, then number who didn't like any of the above music types is

$$n\big((R \cup C \cup E)^c\big) = 68 - 66 = 2.$$

More Inclusion-Exclusion

Example In a survey of 70 students on Movie preferences, the students were asked whether they liked the movies "The Breakfast Club" and "Ferris Bueller's Day Off". (All students had seen both movies and the only options for answers were like/dislike.) 50 of the students said they liked "The Breakfast Club" and 25 of them said they didn't like "Ferris Bueller's Day Off". All students liked at least one of the movies.

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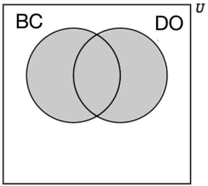

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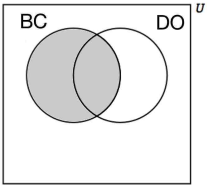
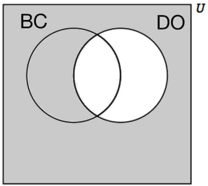
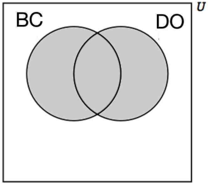


50 Figure 2



25 Figure 3

More Inclusion-Exclusion



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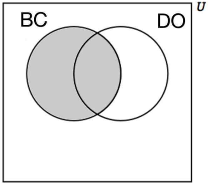

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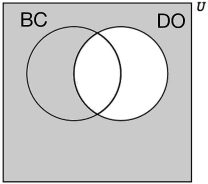

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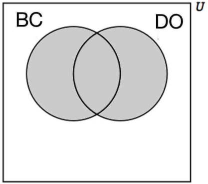

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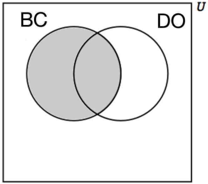

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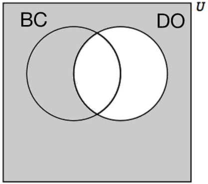

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Since everyone liked at least one movie, $n(U) = 70$.

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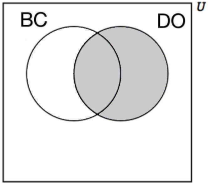

45

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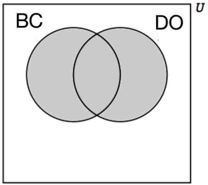

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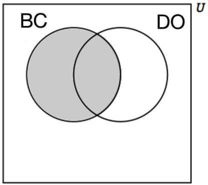

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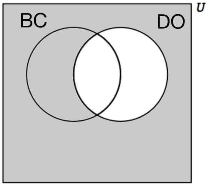

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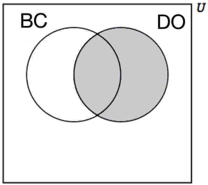
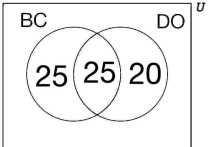


45

Fill in the Venn diagram.



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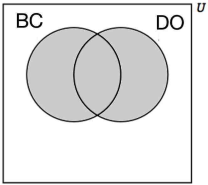

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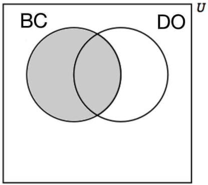

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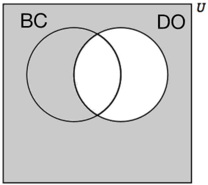

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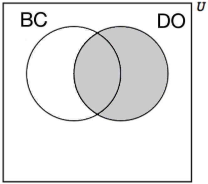

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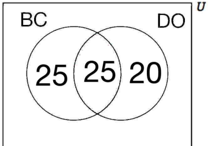

For part (a), 25

More Inclusion-Exclusion

Example In a survey of a group of 70 movie-goers, 62 liked the movie “Catching Fire”, 42 liked the movie “Divergent” and 39 liked both movies.

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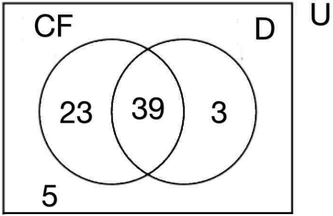

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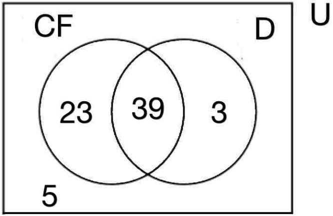

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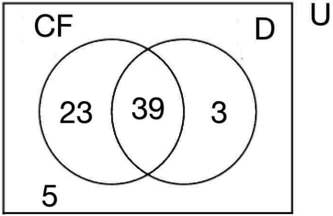

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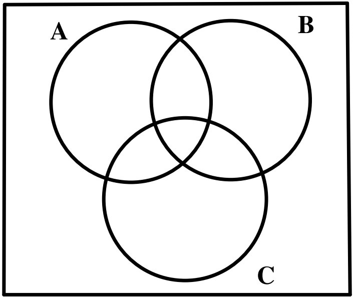
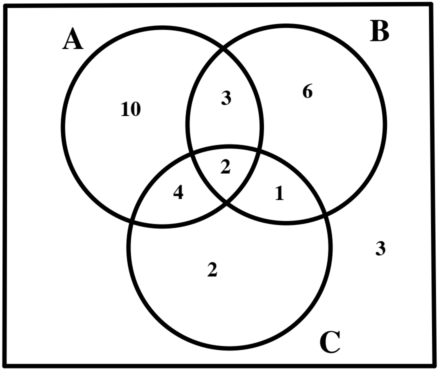
Venn Diagrams of 3 sets

A Venn diagram of 3 sets divides the universal set into 8 non-overlapping regions. We can sometimes use partial information about numbers in some of the regions to derive information about numbers in other regions or other sets.



Venn Diagrams of 3 sets

Example The following Venn diagram shows the number of elements in each region for the sets A, B and C which are subsets of the universal set U.



Find the number of elements in each of the following sets:

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(c) $A \cap B$

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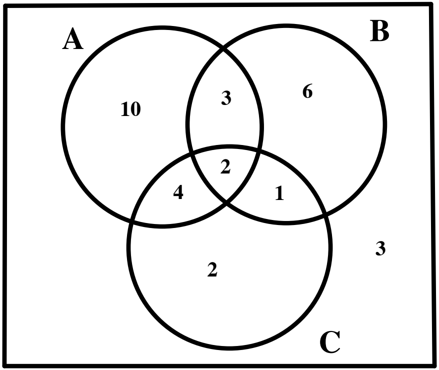

Find the number of elements in each of the following sets:

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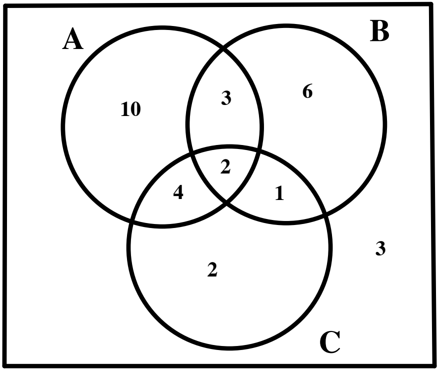

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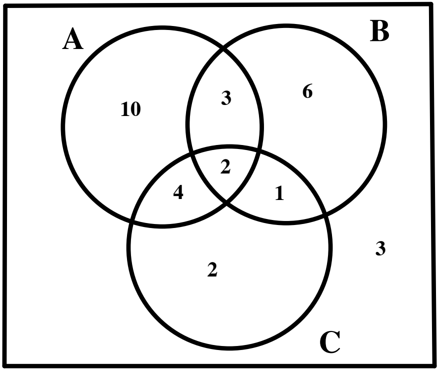

Find the number of elements in each of the following sets:

(a) $A \cap B \cap C$ 2

(b) B' $3 + 2 + 4 + 10 = 19$

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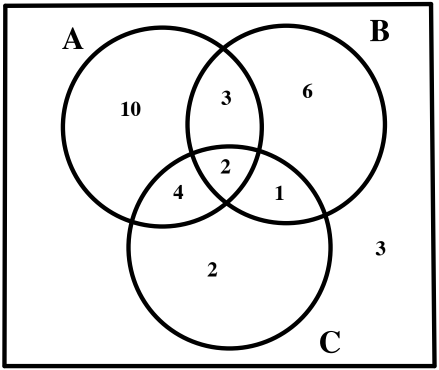

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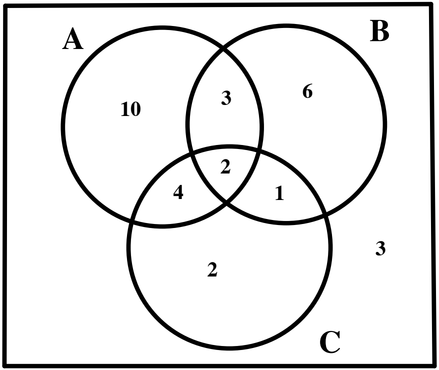

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(e) $B \cup C$ $9 + 3 + 6 = 18$

Venn Diagrams of 3 sets

Example In a survey of a group of 68 Finite Math students, 62 liked the movie “The Fault in our Stars”, 42 liked the movie “The Spectacular Now” and 55 liked the movie “The Perks of Being a Wallflower”. 32 of them liked all 3 movies, 39 of them liked both “The Fault in Our Stars” and “The Spectacular Now”, 35 of them liked both “The Spectacular Now” and “The Perks of Being a Wallflower” and 49 of them liked both “The Fault in Our Stars” and “The Perks of Being a Wallflower”. Represent this information on a Venn Diagram.

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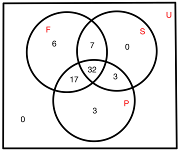

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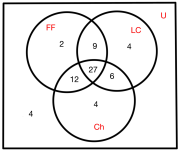

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$n(U) = 68$; $n(V) = 64$; $n(L) = 50$; $n(K) = 36$;
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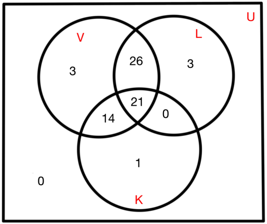
$n(V \cap L \cap K) = 21$.



Old Exam questions for Review

1 In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

(a) 8 (b) 10 (c) 9 (d) 12 (e) 11

Old Exam questions for Review

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(a) 8 (b) 10 (c) 9 (d) 12 (e) 11

$n(U) = 30$; $n(R) = 15$; $n(S) = 13$; $n(C) = 13$.
$n(R \cap S) = 5$; $n(C \cap S) = 8$; $n(R \cap C) = 9$.
$R \cap C \cap S = 5$.

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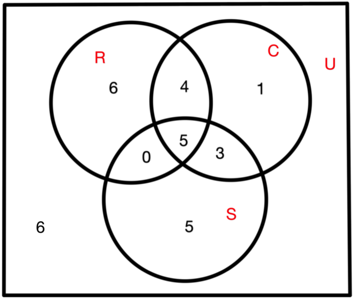

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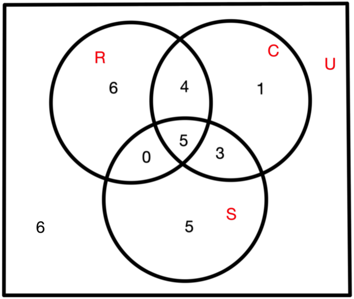

Answer is $6 + 5 = 11$ or (e)

Old Exam questions for Review

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 10 (b) 15 (c) 4 (d) 0 (e) 2

Old Exam questions for Review

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 10 (b) 15 (c) 4 (d) 0 (e) 2

$n(U) = 50$; $n(J) = 25$; $n(B) = 20$; $n(S) = 15$.
$n(B \cap J) = 10$; $n(B \cap S) = 5$; $n(J \cap S) = 7$;
$n(B \cap J \cap S) = 2$;

Old Exam questions for Review

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

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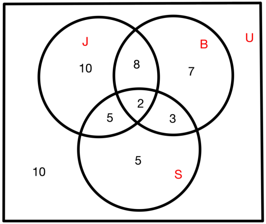

Old Exam questions for Review

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(a) 10 (b) 15 (c) 4 (d) 0 (e) 2

$n\,(U) = 50$; $n\,(J) = 25$; $n\,(B) = 20$; $n\,(S) = 15$.
$n\,(B \cap J) = 10$; $n\,(B \cap S) = 5$; $n\,(J \cap S) = 7$;
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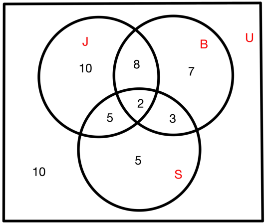
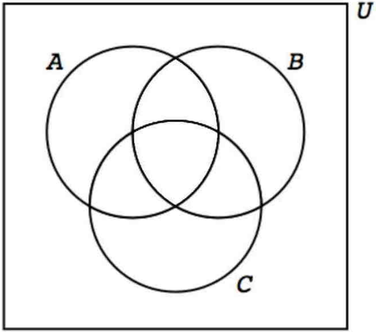


Answer is 10 or (a)

Worked homework problem

Here is an example of a type of problem from the homework. Given 3 subsets A, B and C of a universal set U, suppose $n(U) = 68$; $n(A \cup B \cup C) = 64$; $n(A) = 50$; $n(B) = 49$; $n(C) = 46$. $n(A \cap B) = 39$; $n(C \cap B) = 33$; $n(A \cap C) = 36$. Fill in the Venn diagram.



Worked homework problem

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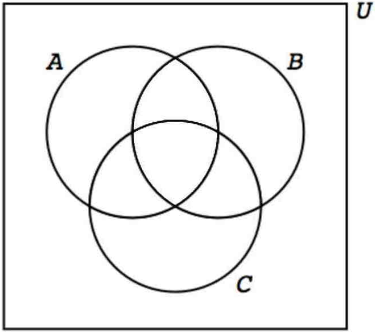
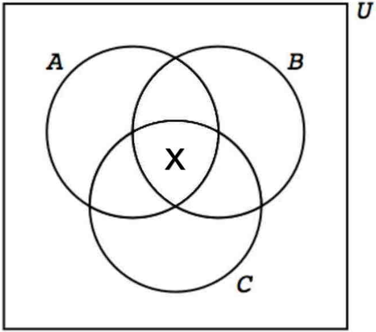


We do not know $n(A \cap B \cap C)$ or this would just be another example of earlier problems.

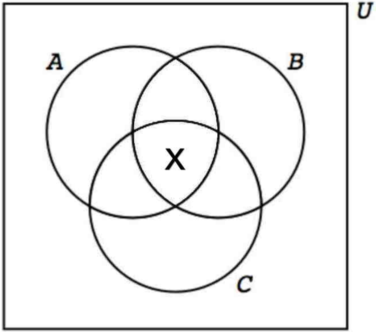
Worked homework problem

Denote $n\,(A \cap B \cap C)$ by x.



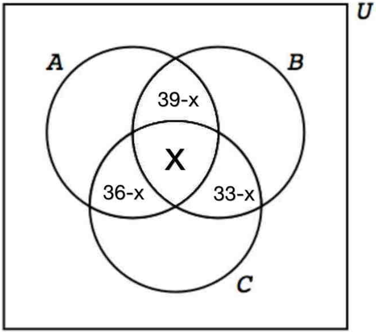
Worked homework problem

Denote $n\,(A \cap B \cap C)$ by x.



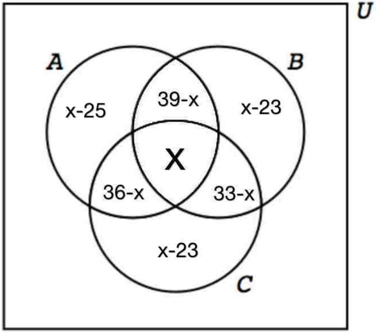
Worked homework problem

Now work out the double intersections: $n(A \cap B) = 39$; $n(C \cap B) = 33$; $n(A \cap C) = 36$.



Worked homework problem

Now work out the sets: $n(A) = 50$; $n(B) = 49$; $n(C) = 46$.



For example, if y denotes the part of A outside of $B \cup C$,

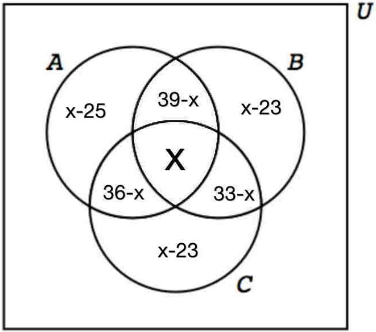
$$50 = y + (39 - x) + (36 - x) + x = y + 75 - x$$

so

$$y = x - 25$$

The others are similar.

Worked homework problem



Since

$$64 = n(A \cup B \cup C) = x + (39 - x) + (36 - x) + (33 - x) + (x - 25) + (x - 23) + (x - 23) = x + (108 - 71) = x + 37$$

Hence $n((A \cup B \cup C)^c) = 68 - 64 = 4.$

Worked homework problem