

Part 1, MULTIPLE CHOICE, 5 Points Each

1 Consider the following sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 4, 6, 8, 10\}$$

$$B = \{1, 2, 3, 4, 5\}.$$

Then $A \cap B'$ is the set

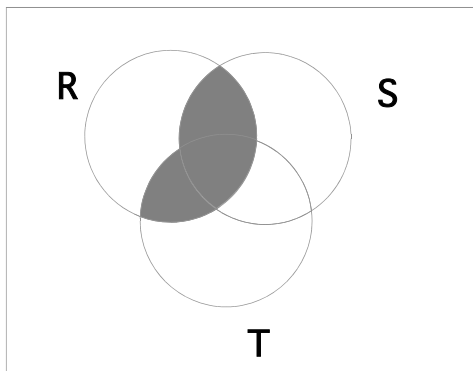
- (a) \emptyset (b) $\{1, 3, 5, 6, 7, 8, 9, 10\}$ (c) $\{6, 8, 10\}$ (d) $\{7, 9\}$ (e) U

2 A survey of 65 students revealed that 40 of them liked Rap, 20 of them liked Alternative Music, 15 of them liked Rap and Techno, 12 of them liked Techno and Alternative music, 5 of them like Alternative music only and 10 of them like all three types of music.

Assume that every student likes at least one of the music types. The number of students who like Techno is

- (a) 50 (b) 30 (c) 23 (d) 35 (e) none of the above

3 Which of the following sets is represented by the shaded region in the Venn diagram below?



- (a) $R \cap T$ (b) $R \cap (S \cup T)$ (c) $R \cup (S \cap T)$ (d) $R \cap (S \cup T)'$ (e) $R \cup S \cup T$

4 Consider the following sets:

$$U = \{\text{All students at Notre Dame}\}$$

$$A = \{\text{All female students at Notre Dame}\}$$

$$B = \{\text{All Students under 20 years of age at Notre Dame}\}$$

Then $(A \cup B)'$ is the set (Hint: you may want to use one of DeMorgan's Laws)

- (a) All students at ND who are male OR at least 20 years of age
(b) All male students at ND who are at least 20 years of age
(c) All male students at ND who are under 20 years of age
(d) All female students at ND who are under 20 years of age
(e) All male students at ND

5 In how many ways can First, Second and third prize be awarded in an art contest with 14 entries, if no prize can be shared?

(a) 14^3

(b) $C(14, 3)$

(c) $P(14, 3)$

(d) $14!$

(e) 3^{14}

6 How many four-letter words (including nonsense words) can be made from the letters of the word
IRRITAINMENT
assuming that letters cannot be repeated.

(a) 7^4

(b) 12^4

(c) $\frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1}$

(d) $\frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}$

(e) $7 \cdot 6 \cdot 5 \cdot 4$

7 Coach Brown has 16 Volleyball players on her squad. She wants to choose 6 to go on court at the beginning of tonight's game and 4 substitutes. In how many ways can she do this?

- (a) $C(16, 10)$ (b) $\frac{C(16,6) \cdot C(10,4)}{2 \cdot 1}$ (c) $C(16, 6) \cdot C(10, 4)$ (d) $P(16, 6)P(10, 4)$ (e) $16!$

8 Jason is about to compete in a race in the An Tostal Festival. He can hop or roll from the start of the race to point A, he can skip, run sideways or run backwards from point A to Point B and he can crawl, or walk on his heels from point B to the finish line. In how many ways can Jason complete the course?

- (a) 12 (b) 7 (c) 3 (d) 14 (e) 5

9 Charles has 1000 songs on his ipod. He is about to go jogging and wants to choose an ordered list of 15 songs to listen to while he is jogging. How many possible lists of 15 songs can he make from the 1000 songs on his ipod.

(a) $P(1000, 15)$

(b) $1000!$

(c) $15!$

(d) 1000

(e) 1000^{15}

10 Suppose an experiment consists of tossing a coin 5 times and observing the sequence of heads and tails. How many different outcomes have at least one head?

(a) 2^5

(b) 5^2

(c) $C(5, 1)$

(d) $C(5, 0) + C(5, 1)$

(e) $2^5 - C(5, 0)$

Part II, PARTIAL CREDIT, 10 Points Each.

Show all of your work for credit

11 (a) Let S and T be sets. Give an equation relating $n(S \cup T)$, $n(S)$, $n(T)$ and $n(S \cap T)$.

(b) If A is a subset of a universal set U , give an equation relating $n(A)$, $n(A')$ and $n(U)$.

(c) If a set, X , has 10 elements, how many subsets does X have?

12 Recall that a poker hand consists of a sample of 5 cards drawn from a deck of 52 cards. The deck has 13 spades, 13 clubs, 13 diamonds and 13 hearts.

(a) How many poker hands have 3 diamonds and 2 spades?

(b) Recall also that a deck of 52 cards can be divided into 13 denominations, (two's through aces), with 4 cards in each denomination. How many poker hands have 3 cards from one denomination and 2 from another? (a house)

(c) How many poker hands are there with 4 Aces?

13 The Notre Dame Squash club has 15 members.

(a) If the club wishes to organize a round robin for its members, (every player plays every other player exactly once), how many matches must be played?

(b) In a survey of the 15 club members it was found that 10 liked to play with Dunlop racquets, 6 liked to play with Black Knight racquets and 2 didn't like to play with either brand. How many of the club members liked to play with both Black Knight and Dunlop racquets?

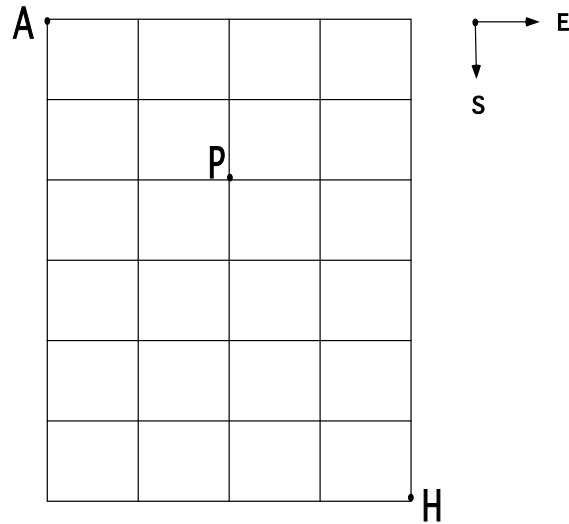
14 An urn contains 10 numbered red balls and 8 numbered blue balls. A random sample of size 5 is drawn from the urn.

(a) How many such samples can be drawn?

(b) How many of the above samples consist of 2 red balls and 3 blue balls?

(c) How many samples of size five have at least one red ball in them?

15 On arriving at LAX airport (at A), in El Pueblo de Nuestra Senora la Reina de los Angeles de Porciuncula, Arnold takes a cab to his home (at H). The grid shown below gives a street map of the area.



(a) If the cab driver always travels south or east (no backtracking), how many routes can he take from the airport at A to Arnold's home at H?

(b) How many of those routes pass through the intersection at P?

(c) If the cab driver knows that there are roadworks at P, how many routes can he take that do not pass through P?