

Part 1, MULTIPLE CHOICE, 5 Points Each

1 An experiment consists of rolling a pair of dice and observing the uppermost faces. The sample space for this experiment consists of 36 outcomes listed as pairs of numbers:

$$S = \{(1, 1), (1, 2), \dots, (6, 6)\}$$

Let E be the event that both faces are even and F the event that both faces add to 6. Which of the following statements is true?

- (a) E and F are mutually exclusive (b) $E \cap F = \{(4, 2), (2, 4)\}$ (c) $E \cup F = S$
(d) $E \cup F' = S$ (e) $E \cap F' = \emptyset$

2 Let E and F be events where $\Pr(E') = \frac{1}{4}$, $\Pr(F) = \frac{1}{4}$, and $\Pr(E \cap F) = \frac{1}{8}$. Find $\Pr(E \cup F)$.

- (a) $\frac{7}{8}$ (b) $\frac{5}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$ (e) 1

3 The odds in favor of the horse “Crackerjack” winning the Melbourne cup horse race are 2:5. What is the probability “Crackerjack” will win?

(a) $2/10$

(b) $2/5$

(c) $2/25$

(d) $2/7$

(e) $1/7$

4 A box, ready for shipment, contains 20 lightbulbs, 4 of which are defective. An inspector selects a sample of 3 lightbulbs from the box. If the inspector finds at least one defective lightbulb among those sampled, the box will not be shipped, otherwise the box will be shipped. What is the probability that this box will pass the inspection and be shipped?

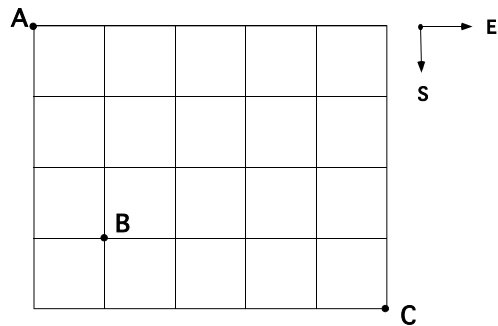
(a) $\frac{C(20,3)}{P(20,3)}$

(b) $\frac{C(4,3) \cdot C(16,3)}{C(20,3)}$

(c) $\frac{C(16,3)}{C(20,3)}$

(d) $\frac{C(4,3)}{C(20,3)}$

(e) $1 - \frac{C(4,3)}{C(20,3)}$



The above is a map of the roads in a country town. A motorist travels from A to C (traveling East or South only). If the motorist is equally likely to choose any of the routes from A to C, what is the probability that their car will pass through B?

- (a) $\frac{1}{7}$ (b) $\frac{11}{63}$ (c) $\frac{4}{21}$ (d) $\frac{13}{63}$ (e) $\frac{10}{63}$

6 In tossing a fair die, we observe the uppermost face. Let E be the event “an odd number occurs” and let F be the event “a number greater than 3 occurs”. What is $P(E|F)$?

- (a) $2/3$ (b) $1/2$ (c) $1/3$ (d) 0 (e) 1

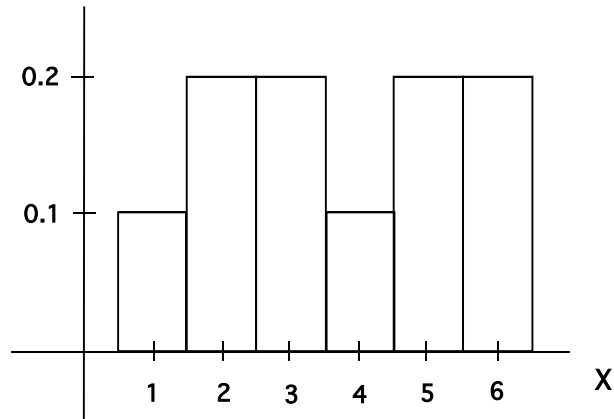
7 A new piece of electronic equipment has five components. the probability of failure within a year is 0.1 for each component. Assuming that the failure of the various components are independent of each other, what is the probability that no component will fail in the first year?

- (a) $(0.1)^5$ (b) $1 - (0.1)^5$ (c) $1 - (0.9)^5$ (d) $(0.9)^5$ (e) 0.1

8 A magician's hat contains 3 rabbits, a squirrel and a groundhog. The magician pulls animals out of her hat at random, stopping when she runs out of animals. What is the probability that the third animal she pulls out of her hat is a groundhog?

- (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{3}{20}$ (e) $\frac{1}{4}$

9 The following is a histogram for the probability distribution of a random variable X.



What is $P(X \leq 3)$?

- (a) 0.4 (b) 0.6 (c) 0.8 (d) 0.2 (e) 0.5

10 A fair die is rolled 10 times. What is the probability that the number on its top face was 5 or higher (i.e. either 5 or 6) on exactly 7 of the rolls?

- (a) $C(10, 7) \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^3$ (b) $P(10, 3) \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3$ (c) $C(10, 3) \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^7$
- (d) $P(10, 7) \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^3$ (e) $P(10, 7) \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3$

Part II, PARTIAL CREDIT, 10 Points Each.

Show all of your work for credit

11(10 Pts) An experiment consists of rolling two dice, a six sided die (with sides labeled 1-6) and a twelve sided die (with sides labeled 1-12). The pair of numbers on the uppermost faces of the dice are observed. Both dice are fair, that is all of their sides are equally likely to face upwards on a single roll of the die.

- (a) How many outcomes are in the sample space for this experiment?

- (b) Let E be the event that you observe a 6 on the twelve sided die, what is the probability of E ?

- (c) Let X denote the sum of the numbers on the uppermost faces, what are the possible values of the random variable X ?

- (d) What is the probability that X takes the value 6, i.e. $Pr(X = 6)$?

12(10 Pts) Each person in a group of 5 people chooses a number (secretly) between 1 and 20 (inclusive). When the numbers are revealed, they put them on a list.

(a) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are allowed?

(b) How many lists of five numbers can be made using the numbers 1-20 (inclusive), if repetitions are not allowed?

(c) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that all five numbers will be different?

(d) If five people each choose a number at random (and independently) from the numbers 1-20 (inclusive), what is the probability that at least two of the numbers will be the same?

13(10 Pts.) A group of 400 students at a small college were studied and information regarding gender and color blindness status was collected from each individual. The results of the study are recorded in the following table:

		Gender	
		Male	Female
Color Blindness status	Yes	10	4
	No	190	196

Let C denote the event that an individual selected at random from the group is color blind and

let M denote the event that an individual selected at random from the group is male.

(a) What is the probability that a randomly selected individual from the group is color blind? that is, what is $P(C)$?

(b) Given that a male is selected at random from this group, what is the probability that he is color blind?

that is, what is $P(C|M)$?

(c) Are the events C and M independent?

Give a reason for your answer.

(d) Are the events C and M Mutually Exclusive?

Give a reason for your answer.

15(10 points) The rules of a carnival game are as follows:

- You pay \$1 to play the game.
- The game attendant then flips a coin at most 4 times.
- As soon as the game attendant gets 2 heads or 3 tails, he stops flipping the coin.
- If the game attendant gets 2 heads, he gives you \$2 (you win).
- If the game attendant gets 3 tails, he gives you nothing (you lose).

(a) Draw a tree diagram representing the possible outcomes of the game.

(b) What is the probability that you win?

(c) Let X denote the earnings for this game. What are the possible values for X ?

(d) Give the probability distribution of X .