**1** Let $U = \{\text{all students at Notre Dame}\}$.

$$F = \{\text{All students at Notre Dame who like to play football}\}$$

$$B = \{\text{All students at Notre Dame who like to play basketball}\}$$

$$S = \{\text{All students at Notre Dame who like to play soccer}\}.$$

Which of the following sets is the set

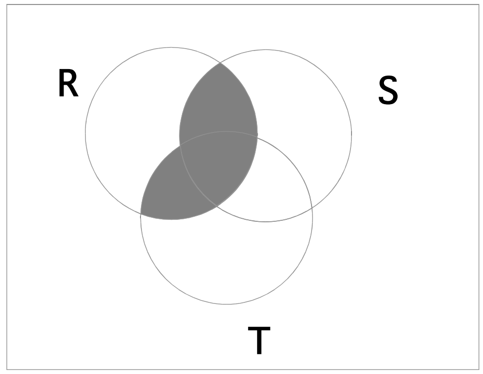
$$\{\text{All students at Notre Dame who like to play basketball and soccer but not football}\}.$$

$(a)$ $(B \cap S)$ $(b)$ $(B \cup S) \cap F'$ $(c)$ $(B \cap S) \cap F'$ $(d)$ $(B \cup S)$ $(e)$ $(B \cup S)' \cap F$

**2** In a group of 70 circus performers, 40 are gymnasts (G), 35 are acrobats (A) and 20 are both acrobats and Gymnasts. How many of the circus performers are neither gymnasts nor acrobats?

$(a)$ 20 $(b)$ 5 $(c)$ 6 $(d)$ 10 $(e)$ 15

**3** Which of the following describes the shaded region in the Venn diagram below?




$(a)$ $(T \cup S) \cap R'$ $(b)$ $(T \cup S)' \cap R$ $(c)$ $(S \cup T) \cap R$ $(d)$ $(T \cap S) \cap R'$ $(e)$ $(S \cap T)' \cap R$

**4** How many three letter words, including nonsense words, can be made from the letters of the word

INDIANA

assuming that each word may NOT have repeated letters.

$(a)$ $7 \cdot 6 \cdot 5$ $(b)$ $4 \cdot 3 \cdot 2$ $(c)$ $7^3$ $(d)$ $4^3$ $(e)$ $3^4$

**5** In how many ways can you choose a hall council consisting of a President, a Vice President, a Secretary and a Treasurer from a pool af 25 applicants, if all of the applicants are eligible for every post and nobody is allowed to hold two posts.

$(a)$ 150 $(b)$ $C(25,4)$ $(c)$ $25^4$ $(d)$ $P(25,4)$ $(e)$ 75

**6** Recall that a Poker hand consists of a sample of 5 cards drawn from a deck of 52 cards, with 13 spades, 13 diamonds, 13 hearts and 13 clubs. How many such hands consist of 2 diamonds and 3 hearts?

$(a)$ $C(13,2)C(13,3)$ $(b)$ $C(13,2)C(39,3)$ $(c)$ $C(52,5)-C(13.2)$

$(d)$ $C(13,2)$ $(e)$ $13 \cdot C(4,3) \cdot 12 \cdot C(4,2)$

**7** An experiment consists of flipping a coin 7 times and the sequence of heads and tails is noted. How many outcomes have more heads than tails?

$(a)$ $C(7,4)-C(7,3)$ $(b)$ $2^4+2^5+2^6+2^7$ $(c)$ $C(7,4)$

$(d)$ $2^7-C(7,3)$ $(e)$ $C(7,4)+C(7,5)+C(7,6)+C(7,7)$

**8** A group of students studying for a final exam order a pizza from "Luigis Express Pizza Service". Let $E$ be the event that the students have to wait more than 30 minutes for the pizza to be delivered. Let $F$ be the event that the students have to wait less than 20 minutes for the pizza to be delivered. If $Pr(E) = .5$, which of the following is true?

(a) The pizza will certainly arrive within 30 minutes

(b) $Pr(F) = 1$

(c) $E \cap F$ is the event that the students have to wait more than 20 and less than 30 minutes for their pizza to arrive.

(d) E and F are mutually exclusive

(e) None of the above

**9** If $Pr(E) = .3, \quad Pr(F) = .8$ and $Pr(E \cup F) = .9$, find $Pr(E \cap F)$.

$(a)$ 0.3 $(b)$ 0.2 $(c)$ 0.1 $(d)$ 0.24 $(e)$ 0.27

**10** An urn contains 4 pink and 6 blue balls. A sample of size five is drawn from the urn. What is the probability that this sample contains exactly 3 pink and 2 blue balls?

$(a)$ $\dfrac{C(4,3)}{2^5}$ $(b)$ $\dfrac{C(4,3)}{C(10,5)}$ $(c)$ $\dfrac{C(6,2)}{C(10,5)}$ $(d)$ $C(6,2)C(4,3)$ $(e)$ $\dfrac{C(6,2)C(4,3)}{C(10,5)}$

**11** Of students at a certain college, 95% regularly attend football games, 25% are freshmen and 20% are freshmen who regularly attend football games. A student is selected at random. What is the probability that the student regularly attends football games given that they are a freshman?

$(a)$ 0.8 $(b)$ 0.2 $(c)$ $\dfrac{2}{95}$ $(d)$ 0.25 $(e)$ 1

**12** David is taking a quiz with 6 multiple choice questions. Each question has 5 possible answers. If David gives a random guess for the answer to each question, what is the probability that he will get all 6 correct?

$(a)$ $\dfrac{C(5,1)}{2^6}$ $(b)$ $\dfrac{1}{5^6}$ $(c)$ $\dfrac{C(6,5)}{2^6}$ $(d)$ $\dfrac{1}{6^5}$ $(e)$ $\dfrac{C(5,1)^6}{2^6}$

**13** The rules of a carnival game are as follows:
First you roll a die.
If the number on the uppermost face of the die is a 1 or a 6, you toss a coin.
if the coin shows a head, you win one dollar, if the outcome on the coin is a tail, you lose one dollar.
If the number on the uppermost face of the die is a 2, 3, 4, or 5, you draw a card from a deck of 52.
If the card is an ace, you win one dollar, if the card is not an ace, you lose one dollar.
What is the probability that you win one dollar, if you play this game (A tree diagram may help)?

$(a)\ \frac{8}{156}$ $\quad(b)\ \frac{122}{156}$ $\quad(c)\ \frac{1}{6}$ $\quad(d)\ \frac{1}{3}$ $\quad(e)\ \frac{34}{156}$

**14** At the beginning of Spring practice, 20 Notre Dame footballers are asked to perform a vertical leap as part of a fitness test. The heights in inches from the ground and the frequencies are recorded in the following table:

| height | 27 | 29 | 30 | 31 | 32 | 33 | 34 | 36 |
|---|---|---|---|---|---|---|---|---|
| frequencies | 1 | 2 | 3 | 3 | 3 | 4 | 3 | 1 |

Find the average height of the vertical leaps performed by the football players in the above group.

$(a)$ 31.5 $\quad(b)$ 31 $\quad(c)$ 31.7 $\quad(d)$ 33 $\quad(e)$ 20

**15** Find the variance, $\sigma^2 = Var(X)$, of the random variable $X$, whose distribution is given in the table below.

| k | Pr(X = k) |
|---|---|
| 1 | 0.1 |
| 2 | 0.2 |
| 3 | 0.4 |
| 4 | 0.2 |
| 5 | 0.1 |

$(a)$ 1.2 $(b)$ 10 $(c)$ 3 $(d)$ 2.4 $(e)$ 1

**16** If $Z$ is a standard normal random variable, what is

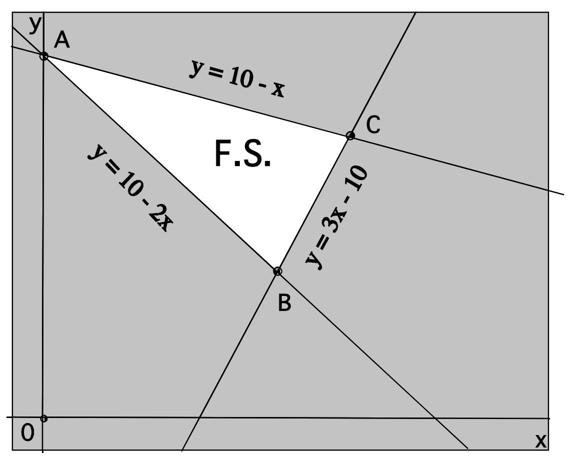
$$Pr(-1.5 \leq Z \leq 1)?$$

$(a)$ .8413 $(b)$ .7745 $(c)$ .0668 $(d)$ .1587 $(e)$ .2255

**17** At Katy's shoe store, the number of shoes sold in one day is normally distributed with mean $\mu = 100$ pairs and standard deviation $\sigma = 20$ pairs. What is the probability that 110 or more pairs are sold in one day?

$(a)$ 0.2743 $(b)$ 0.6915 $(c)$ 0.9772 $(d)$ 0.0228 $(e)$ 0.3085

**18** What is the <u>maximum</u> of the objective function $3x + 5y$ on the feasible set shown below.




$(a)$ 35 $(b)$ 40 $(c)$ 30 $(d)$ 22 $(e)$ 50

**19** The Notre Dame Math Lovers society will make t-shirts and sweatshirts for its annual fundraiser. It takes 1 hour to cut the material for a t-shirt and 1 hour to cut the material for a sweatshirt. It takes 1 hour to complete the sewing necessary to make a t-shirt and 2 hours to complete the sewing necessary for a sweatshirt. It takes 2 hours to finish the t-shirt and 3 hours to finish the sweatshirt. Connor, will cut the material and has 40 hours available for this task. Shayln, will do the sewing and has 65 hours available to do so. Ansley will finish the t-shirts and sweatshirts and has 100 hours to spend finishing them. The society will make a profit of \$5 on each t-shirt made and a profit of \$8 on each sweatshirt made. Let x denote the number of t-shirts that will be made and let y denote the number of sweatshirts to be made. Which of the following sets of inequalities describe the constraints for production and the profit function?

(a)
$$\begin{gathered} x \geq 0, \quad y \geq 0 \\ x + y \leq 40 \\ x + 2y \leq 65 \\ 2x + 3y \leq 100 \\ \text{Profit function: } 5x + 8y \end{gathered}$$

(b)
$$\begin{gathered} x \geq 0, \quad y \geq 0 \\ x + y \leq 40 \\ 2x + y \leq 65 \\ 3x + 2y \leq 100 \\ \text{Profit function: } 8x + 5y \end{gathered}$$

(c)
$$\begin{gathered} x \geq 0, \quad y \geq 0 \\ x + y \leq 65 \\ x + 2y \leq 40 \\ 2x + 3y \leq 100 \\ \text{Profit function: } 5x + 8y \end{gathered}$$

(d)
$$\begin{gathered} x \geq 0, \quad y \geq 0 \\ x + y \geq 40 \\ x + 2y \geq 65 \\ 2x + 3y \geq 100 \\ \text{Profit function: } 5x + 8y \end{gathered}$$

(e)
$$\begin{gathered} x \geq 0, \quad y \geq 0 \\ 2x + y \leq 40 \\ x + 2y \leq 65 \\ 2x + 3y \leq 100 \\ \text{Profit function: } 8x + 5y \end{gathered}$$

**20** Let $A =$

$$\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$$

Which of the following gives the entry in the 2nd row and 1st column of $A^{-1}$?

$(a)$ $-1$ $(b)$ $3$ $(c)$ $1$ $(d)$ $-2$ $(e)$ $\frac{1}{3}$

**21** Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 \\ 5 & 0 \\ 0 & 1 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

Calculate $(A - B) \cdot C$.

$(a)$ $\begin{pmatrix} 0 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $(b)$ $\begin{pmatrix} -1 & 1 \\ -2 & 1 \\ 0 & 1 \end{pmatrix}$ $(c)$ $\begin{pmatrix} 1 & 0 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$

$(d)$ $\begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}$ $(e)$ $\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$

**22** Let

$$C = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 2 & 4 \end{pmatrix}, \qquad D = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}.$$

Find the entry in the second row and first column of the matrix $C \cdot D$.

$(a)$ 10 $(b)$ 4 $(c)$ 7 $(d)$ 17 $(e)$ 0

**23** Let

$$A = \begin{pmatrix} 5 & 2 \\ 1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 5 & 2 \\ 1 & 0 \\ 2 & 1 \end{pmatrix}, \qquad D = \begin{pmatrix} 2 & 1 & 5 \end{pmatrix}.$$

Which of the following statements is true?

(a) $A^{-1}$ does not exist.

(b) $(B - C) \cdot A$ does not exist.

(c) $D \cdot C$ does not exist.

(d) $B \cdot A$ does not exist.

(e) $C \cdot B$ does not exist.

**24** The following matrix is the payoff matrix for the row player in a zero-sum game:

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & -2 \\ -1 & 0 & 1 \end{pmatrix}$$

The payoff matrix has a saddle point; where is it?

$(a)$ Row 3, Col 1 $\quad$ $(b)$ Row 1, Col 3 $\quad$ $(c)$ Row 2, Col 3 $\quad$ $(d)$ Row 1, Col 1 $\quad$ $(e)$ Row 2, Col 2

**25** Roadrunner (R) and Coyote (C) play a game. They each have 4 cards, numbered 1, 2, 3 and 4. They each display one card simultaneously. If both numbers are even Coyote gives Roadrunner \$1. If both numbers are odd, Roadrunner gives Coyote \$1. If the numbers are neither both even nor both odd, the creature displaying the higher number receives \$1 from the other creature. Which of the following payoff matrices gives the payoff matrix for Roadrunner for this game?

$(a)$

| | | | | C | |
|---|---|---|---|---|---|
| | Card # | 1 | 2 | 3 | 4 |
| | 1 | −1 | 1 | 1 | 1 |
| | 2 | −1 | 1 | 1 | 1 |
| $R$ | 3 | −1 | −1 | 1 | 1 |
| | 4 | −1 | −1 | −1 | −1 |

$(b)$

| | | | | C | |
|---|---|---|---|---|---|
| | Card # | 1 | 2 | 3 | 4 |
| | 1 | 1 | 1 | −1 | −1 |
| | 2 | 1 | 1 | −1 | −1 |
| $R$ | 3 | 1 | 1 | 1 | −1 |
| | 4 | 1 | 1 | 1 | 1 |

$(c)$

| | | | | C | |
|---|---|---|---|---|---|
| | Card # | 1 | 2 | 3 | 4 |
| | 1 | 1 | 0 | 0 | 1 |
| | 2 | 0 | 1 | 2 | 3 |
| $R$ | 3 | 0 | 0 | 1 | 2 |
| | 4 | 1 | 1 | −1 | −1 |

$(d)$

| | | | | C | |
|---|---|---|---|---|---|
| | Card # | 1 | 2 | 3 | 4 |
| | 1 | −1 | 0 | 1 | 1 |
| | 2 | 0 | −1 | 1 | −1 |
| $R$ | 3 | 0 | 0 | −1 | −1 |
| | 4 | 1 | 1 | −1 | −1 |

$(e)$

| | | | | C | |
|---|---|---|---|---|---|
| | Card # | 1 | 2 | 3 | 4 |
| | 1 | −1 | −1 | −1 | −1 |
| | 2 | 1 | 1 | −1 | 1 |
| $R$ | 3 | −1 | 1 | −1 | −1 |
| | 4 | 1 | 1 | 1 | 1 |

**26** Rat (R) and Cat (C) play a zero-sum game with payoff matrix for Rat given below. What is the optimal pure strategy for Cat for this game?

$$\begin{pmatrix} 1 & 0 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 & 2 \\ 3 & 2 & -1 & 4 & 6 \\ -1 & -2 & 1 & -1 & -2 \\ 0 & 1 & -1 & 0 & -5 \end{pmatrix}$$

$(a)$ Col 1 $(b)$ Col 2 $(c)$ Col 3 $(d)$ Col 4 $(e)$ Col 5

**27** Catman (C) and Robin (R) play a zero-sum game, with payoff matrix for Robin given by

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy $(.8\ .2)$ and Catman plays the mixed strategy $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ What is the expected payoff for Robin for the game?

$(a)$ 1.4 $(b)$ 1.48 $(c)$ 1.6 $(d)$ .5 $(e)$ .8

**28** Suppose the payoff matrix for Robin, in a zero sum game with Catman, is as in the previous problem:

$$\begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

If Robin plays the mixed strategy $(.8\ .2)$, which of the following mixed strategies should Catman play to maximize his (Catman's) expected payoff in the game?

$(a)$ $\begin{pmatrix} .6 \\ .4 \end{pmatrix}$ $(b)$ $\begin{pmatrix} .4 \\ .6 \end{pmatrix}$ $(c)$ $\begin{pmatrix} .3 \\ .7 \end{pmatrix}$ $(d)$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $(e)$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

**29** Rapunzel (R) and Cinderella (C) play a zero-sum game with payoff matrix for Rapunzel given by

$$\begin{pmatrix} 5 & 2 \\ 1 & 3 \end{pmatrix}.$$

Rapunzel wants to find the optimal mixed strategy, assuming that Cinderella always plays the best counterstrategy. Which of the following linear programming problems must she solve:

$(a)$
$$\begin{array}{ll} \text{minimize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & x+5y \quad \geq 1 \\ & 3x+2y \quad \geq 1 \end{array}$$

$(b)$
$$\begin{array}{ll} \text{maximize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & 5x+y \quad \geq 1 \\ & 2x+3y \quad \geq 1 \end{array}$$

$(c)$
$$\begin{array}{ll} \text{minimize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & 5x+y \quad \leq 1 \\ & 2x+3y \quad \leq 1 \end{array}$$

$(d)$
$$\begin{array}{ll} \text{maximize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & x+5y \quad \leq 1 \\ & 3x+2y \quad \leq 1 \end{array}$$

$(e)$
$$\begin{array}{ll} \text{minimize} & x+y \\ \text{constraints} & x \geq 0, \quad y \geq 0 \\ & 5x+y \quad \geq 1 \\ & 2x+3y \quad \geq 1 \end{array}$$

**30** If Rapunzel found that the solution to the linear programming problem for Question 29 was

$$x = \frac{2}{13}, \quad y = \frac{3}{13},$$

what would her optimal mixed strategy be?

$(a)$ $\left(\frac{4}{5}, \frac{1}{5}\right)$ $(b)$ $\left(\frac{2}{5}, \frac{3}{5}\right)$ $(c)$ $\left(\frac{3}{5}, \frac{2}{5}\right)$ $(d)$ $\left(\frac{10}{13}, \frac{3}{13}\right)$ $(e)$ $(0, 1)$

Areas under the Standard Normal Curve

| $z$ | $A(z)$ | $z$ | $A(z)$ | $z$ | $A(z)$ | $z$ | $A(z)$ | $z$ | $A(z)$ |
|---|---|---|---|---|---|---|---|---|---|
| −3.50 | .0002 | −2.00 | .0228 | −.50 | .3085 | 1.00 | .8413 | 2.50 | .9938 |
| −3.45 | .0003 | −1.95 | .0256 | −.45 | .3264 | 1.05 | .8531 | 2.55 | .9946 |
| −3.40 | .0003 | −1.90 | .0287 | −.40 | .3446 | 1.10 | .8643 | 2.60 | .9953 |
| −3.35 | .0004 | −1.85 | .0322 | −.35 | .3632 | 1.15 | .8749 | 2.65 | .9960 |
| −3.30 | .0005 | −1.80 | .0359 | −.30 | .3821 | 1.20 | .8849 | 2.70 | .9965 |
| −3.25 | .0006 | −1.75 | .0401 | −.25 | .4013 | 1.25 | .8944 | 2.75 | .9970 |
| −3.20 | .0007 | −1.70 | .0446 | −.20 | .4207 | 1.30 | .9032 | 2.80 | .9974 |
| −3.15 | .0008 | −1.65 | .0495 | −.15 | .4404 | 1.35 | .9115 | 2.85 | .9978 |
| −3.10 | .0010 | −1.60 | .0548 | −.10 | .4602 | 1.40 | .9192 | 2.90 | .9981 |
| −3.05 | .0011 | −1.55 | .0606 | −.05 | .4801 | 1.45 | .9265 | 2.95 | .9984 |
| −3.00 | .0013 | −1.50 | .0668 | .00 | .5000 | 1.50 | .9332 | 3.00 | .9987 |
| −2.95 | .0016 | −1.45 | .0735 | .05 | .5199 | 1.55 | .9394 | 3.05 | .9989 |
| −2.90 | .0019 | −1.40 | .0808 | .10 | .5398 | 1.60 | .9452 | 3.10 | .9990 |
| −2.85 | .0022 | −1.35 | .0885 | .15 | .5596 | 1.65 | .9505 | 3.15 | .9992 |
| −2.80 | .0026 | −1.30 | .0968 | .20 | .5793 | 1.70 | .9554 | 3.20 | .9993 |
| −2.75 | .0030 | −1.25 | .1056 | .25 | .5987 | 1.75 | .9599 | 3.25 | .9994 |
| −2.70 | .0035 | −1.20 | .1151 | .30 | .6179 | 1.80 | .9641 | 3.30 | .9995 |
| −2.65 | .0040 | −1.15 | .1251 | .35 | .6368 | 1.85 | .9678 | 3.35 | .9996 |
| −2.60 | .0047 | −1.10 | .1357 | .40 | .6554 | 1.90 | .9713 | 3.40 | .9997 |
| −2.55 | .0054 | −1.05 | .1469 | .45 | .6736 | 1.95 | .9744 | 3.45 | .9997 |
| −2.50 | .0062 | −1.00 | .1587 | .50 | .6915 | 2.00 | .9772 | 3.50 | .9998 |
| −2.45 | .0071 | −.95 | .1711 | .55 | .7088 | 2.05 | .9798 | | |
| −2.40 | .0082 | −.90 | .1841 | .60 | .7257 | 2.10 | .9821 | | |
| −2.35 | .0094 | −.85 | .1977 | .65 | .7422 | 2.15 | .9842 | | |
| −2.30 | .0107 | −.80 | .2119 | .70 | .7580 | 2.20 | .9861 | | |
| −2.25 | .0122 | −.75 | .2266 | .75 | .7734 | 2.25 | .9878 | | |
| −2.20 | .0139 | −.70 | .2420 | .80 | .7881 | 2.30 | .9893 | | |
| −2.15 | .0158 | −.65 | .2578 | .85 | .8023 | 2.35 | .9906 | | |
| −2.10 | .0179 | −.60 | .2743 | .90 | .8159 | 2.40 | .9918 | | |
| −2.05 | .0202 | −.55 | .2912 | .95 | .8289 | 2.45 | .9929 | | |