Part 1, MULTIPLE CHOICE, 5 Points Each

1 The probability distribution for a random variable X is given below. What is the variance of X (denoted σ^2 or Var(X))?

k	Pr($X = k$)
-2	.3
-1	.1
0	.2
1	.1
2	.3

(a) 0 (b) 2.6 (c) 10 (d) 2 (e) 1.5

2 Let Z be a random variable with a standard normal distribution. What is $Pr(Z \geq -1)$? (Use the attached tables)

(a) .1587 (b) .5 (c) .8413 (d) .5398 (e) .4602

3 Which of the pairs of values for x and y given below is in the feasible set for the set of inequalities:

$$x + 2y \geq 7$$

$$2x - y \geq 4$$

$$y \geq 0, \quad x \geq 0.$$

(a) $x = 1, \quad y = 3$ (b) $x = 0, \quad y = 4$ (c) $x = 2, \quad y = 1$

(d) $x = 3, \quad y = -2$ (e) $x = 7, \quad y = 1$

4 Which of the following give the co-ordinates of the point of intersection of the two lines:

$$5x - y = 2$$

$$-8x + 4y = 4$$

(a) $x = 3, \ y = 13$ (b) $x = \frac{1}{9}, \ y = \frac{13}{9}$ (c) $x = 1, \ y = 3$

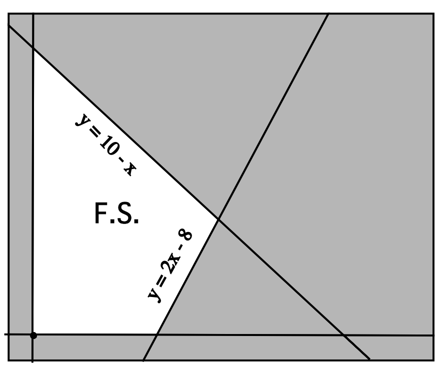
(d) $x = 2, \ y = 8$ (e) $x = 2, \ y = 10$

5 A student spending spring break in Ireland wants to visit Galway and Cork. The student has at most 7 days available and at most 500 euros to spend. Each day spent in Galway will cost 50 euros and each day spent in Cork will cost 60 euros. Let x be the number of days the student will spend in Galway and y, the number of days the student will spend in Cork. Which of the following sets of constraints describe the constraints on the student's time and money for the visits?

(a) $\begin{array}{c} x+y \leq 7 \\ 50x+60y \leq 500 \\ x \geq 0, \quad y \geq 0 \end{array}$ (b) $\begin{array}{c} x+7y \leq 500 \\ 50x+60y \leq 1000 \\ x \geq 0, \quad y \geq 0 \end{array}$ (c) $\begin{array}{c} x+y \leq 7 \\ 60x+50y \leq 500 \\ x \geq 0, \quad y \geq 0 \end{array}$

(d) $\begin{array}{c} x+y \geq 7 \\ 50x+60y \geq 500 \\ x \geq 0, \quad y \geq 0 \end{array}$ (e) $\begin{array}{c} x+y \geq 7 \\ 60x+50y \geq 500 \\ x \geq 0, \quad y \geq 0 \end{array}$

6 Find the maximum value of the objective function $20x + 5y$ on the feasible set shown below.



(a) 50 (b) 180 (c) 80 (d) 140 (e) 90

7 If A and B are the matrices given below, find $2A - 3B$.

$$A = \begin{pmatrix} 1 & 2 & 1 & 4 \\ -1 & 1 & 2 & 1 \\ 1 & -2 & 3 & 6 \end{pmatrix}, \qquad B = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 4 \\ 3 & 0 & -4 & 2 \end{pmatrix}$$

(a) $\begin{pmatrix} 2 & 2 & 1 & 3 \\ -3 & 0 & 2 & -3 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 4 & 2 & 11 \\ 4 & 5 & 4 & 14 \\ 11 & -4 & -6 & 18 \end{pmatrix}$ (c) $\begin{pmatrix} 5 & 4 & 2 & 5 \\ -8 & -1 & 4 & -10 \\ -7 & -4 & 18 & 6 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 2 & 1 \\ 1 & 2 & 2 \\ 4 & -2 & -1 \end{pmatrix}$ (e) $\begin{pmatrix} 5 & 4 & 2 & 5 \\ 4 & 5 & 4 & 14 \\ -2 & -2 & 7 & 4 \end{pmatrix}$

8 Let $A = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$. Which of the following is the matrix A^{-1}?

(a) $\begin{pmatrix} \frac{3}{2} & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 2 \\ \frac{1}{2} & -\frac{3}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}$ (e) $\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -2 & 1 \end{pmatrix}$

9 Rapunzel (R) and Cinderella (C) play a game where they both choose a number between 1 and 4 inclusive. Cinderella then calculates the value of Rapunzel's number minus Cinderella's number and pays Rapunzel that amount in dollars. (If this is a negative number then Cinderella will receive money from Rapunzel.) Which of the following matrices gives the payoff matrix for Rapunzel for this game?

(a)

	Num.	C 1	2	3	4
	1	4	3	2	1
	2	3	4	3	2
R	3	2	3	4	3
	4	1	2	3	4

(b)

	Num.	C 1	2	3	4
	1	0	1	2	3
	2	1	0	1	2
R	3	2	1	0	1
	4	3	2	1	0

(c)

	Num.	C 1	2	3	4
	1	1	2	3	4
	2	2	1	2	3
R	3	3	2	1	2
	4	4	3	2	1

(d)

	Num.	C 1	2	3	4
	1	0	−1	2	−3
	2	−1	0	−1	2
R	3	2	−1	0	−1
	4	−3	2	−1	0

(e)

	Num.	C 1	2	3	4
	1	0	−1	−2	−3
	2	1	0	−1	−2
R	3	2	1	0	−1
	4	3	2	1	0

10 Romeo (R) and Collette (C) play a zero-sum game for which the payoff matrix for Romeo is given by:

	$C1$	$C2$	$C3$	$C4$	$C5$
$R1$	−1	5	9	1	4
$R2$	3	−1	−3	2	7
$R3$	−2	−3	1	9	8
$R4$	1	2	2	0	14

If Romeo always plays R3, which column should Collette play in order to maximize her gain?

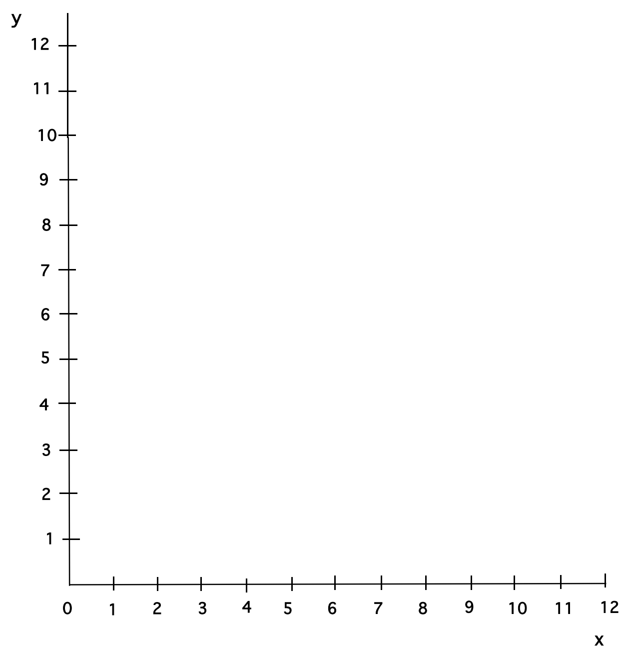
(a) $C1$ (b) $C2$ (c) $C3$ (d) $C4$ (e) $C5$

Part II, PARTIAL CREDIT, (10 Points each)

Show all of your work for credit

11, (a) Graph the feasible set corresponding to the following set of inequalities on the set of axes provided:

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ y + 2x &\leq 8 \\ y + 6x &\geq 12 \end{aligned}$$



(b) Find the vertices of the above feasible set.

(c) Find the <u>maximum</u> of the objective function $5x + 10y$ on the above feasible set.

12, The following are golf scores from 10 rounds of golf for two players, Player A and Player B:

Player A:	91,	95,	96,	94,	95,	92,	93,	99,	97,	98
Player B:	96,	95,	97,	94,	95,	94,	93,	95,	96,	95

The AVERAGE SCORE in each case is 95

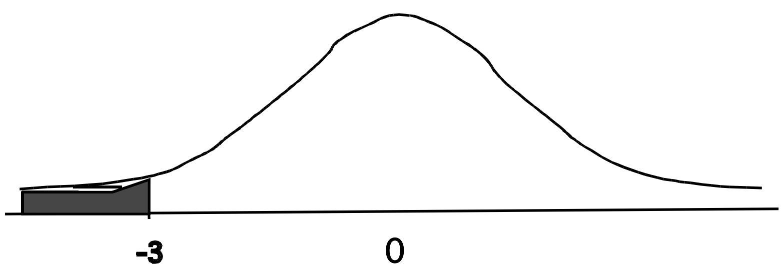
(a) Calculate the population variance, σ^2 for player A.

(b) Calculate the population variance, σ^2 for player B.

(c) Which player has the most consistent scores?

13, (a) The number of cups of regular coffee sold in Starbucks each day is normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 20$. What is the probability that less than 450 cups of regular coffee will be sold at Starbucks today?

(b) The following diagram represents the standard normal curve i.e. the distribution of a standard normal random variable, Z, with mean $\mu = 0$ and standard deviation $\sigma = 1$. What is the area of the shaded region in the diagram?



(c) If Z is a standard normal random variable, i.e. has mean $\mu = 0$ and standard deviation $\sigma = 1$ what is $Pr(Z \geq 3)$? (You could use the symmetry in the above curve if you wanted to be adventurous!)

14, Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 1 & 3 & 1 \\ 0 & 2 & 5 & 1 \end{pmatrix}$$

(a) Calculate the product $A \cdot B$.

(b) Let

$$C = \begin{pmatrix} 1 & 5 \\ 1 & 7 \end{pmatrix}.$$

Which of the following products can be calculated?

$$A \cdot C, \quad C \cdot A, \quad B \cdot C, \quad C \cdot B.$$

(c) What is the product of the following three matrices?

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 1 & 7 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

15 (a) Cat (C) and Rat (R) play a 2-player zero-sum game, where the payoff matrix for Rat is given by the following matrix:

	$C1$	$C2$	$C3$
$R1$	2	1	0
$R2$	-2	0	-1
$R3$	0	-3	-2

What is Rat's optimal fixed(pure) strategy for this game ?

What is Cat's optimal fixed strategy for this game?

Does this pay-off matrix have a saddle point? If so where?

(b) Catherine (C) and Rasputin (R) play a 2-player zero-sum game, where the payoff matrix for Rasputin is given by the following matrix:

	$C1$	$C2$	$C3$
$R1$	-2	1	0
$R2$	-1	0	5

What is Rasputin's optimal fixed(pure) strategy for this game ?

What is Catherine's optimal fixed strategy for this game?

Does this pay-off matrix have a saddle point? If so where?

(c) Catman (C) and Robin (R) play a 2-player zero-sum game, where the payoff matrix for Robin is given by the following matrix:

	$C1$	$C2$	$C3$
$R1$	2	1	4
$R2$	5	-1	3
$R3$	1	2	-5

What is Robin's optimal fixed(pure) strategy for this game ?

What is Catman's optimal fixed strategy for this game?

Does this pay-off matrix have a saddle point? If so where?